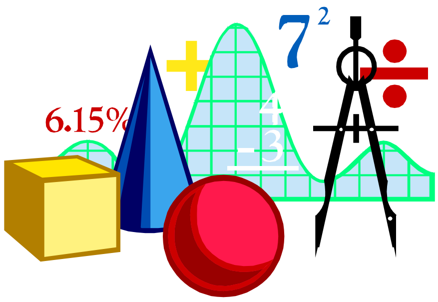
ALGEBRA FOR PREPARATORY TWO FIRST TERM

PREPARED BY Mr. MAHMOUD



Sheet (1)
Important Revision

[1] Complete by writing the following numbers in the form $\frac{a}{b}$ where a & b are two integers in the simplest form, $b \neq 0$:

(1) 0.2 = ……… (2) 0.3 = ………

(3) 25% = ……… (4) |-0.75| = ………

(5) -6 = ……… (6) $1\frac{1}{4}$ = ………

[2] Complete the following:

(1) $\sqrt{25+144}$ = ………………

(2) $\sqrt{0.25}$ = ………………

(3) The standard form of the number 0.00015 is ………………………

(4) The standard form of the number 421×10^3 is ………………………

(5) The sum of the two square roots of each number $2\frac{1}{4}$ = ……………

[3] Choose the correct answer:

(1) $Z^+ \cup \{0\} =$ ……… (Z , N , R , Q)

(2) |-2| + |-4| + |6| = ……… (zero , |-12| , -12 , 6)

(3) $\sqrt{a^2}$ = ……… (a , -a , |a| , ±a)

(4) $\sqrt{100-36}$ = ……… (4 , ±4 , 8 , ±8)

(5) $\frac{\sqrt{25-9}}{\sqrt{25}-\sqrt{9}}$ = ……… (-1 , 1 , 2 , 3)

(6) Which of the following rational numbers lies between $\frac{1}{5}$ and $\frac{2}{5}$? ……………… ($\frac{2}{10}$, $\frac{1}{10}$, 0.3 , -0.3)

(7) The product of the rational number $\frac{a}{b}$ by its additive inverse equals …………… (zero , $\frac{-a}{b}$, $\frac{a^2}{b^2}$, $\frac{-a^2}{b^2}$)

(8) $3^{10} + 3^{10} + 3^{10} =$ ……… (3^{10} , 3^{30} , 9^{10} , 3^{11})

(9) If $a^{-1} = \frac{2}{3}$, then a = ……… ($-\frac{2}{3}$, $\frac{3}{2}$, $-\frac{3}{2}$, 1)

(10) The multiplicative inverse of 5^{-1} is …………… ($\frac{1}{5}$, 5 , -5 , $\frac{-1}{5}$)

[4] Find the value of x in the following equations:

(1) $5x + 3 = 20$

……………………………………………
……………………………………………
……………………………………………
……………………………………………

(2) $5x + 11 = 12$

……………………………………………
……………………………………………
……………………………………………
……………………………………………

(3) $3x + 5 = 1$

……………………………………………
……………………………………………
……………………………………………
……………………………………………

(4) $x + 3 = 7$

……………………………………………
……………………………………………
……………………………………………
……………………………………………

[5] Find the solution set of each of the following equations, where $x \in Q$:

(1) $x^2 + 12 = 21$

………………………………………
………………………………………
………………………………………
………………………………………

(2) $2x^2 - 1 = -9$

………………………………………
………………………………………
………………………………………
………………………………………

(3) $|x| = 2$

………………………………………
………………………………………
………………………………………
………………………………………

(4) $\sqrt{x^2} = 4$

………………………………………
………………………………………
………………………………………
………………………………………

Sheet (2)

The cube root of a rational number

Example (1):

Find the area of a square whose side length 5 cm?

Area = S × S = $S^2 = 5^2 = 25$ cm^2.

The square numbers	The square root
$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$
$\sqrt{x^4} = x^2$	$\sqrt{x^6} = x^3$

Example (2):

Find the volume of a cube whose edge length 5 cm?

V = S × S × S = $S^3 = 5^3 = 125$ cm^3.

The cub numbers	The cube root
$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$
$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^2 = 512$	$\sqrt[3]{512} = 8$
$9^3 = 729$	$\sqrt[3]{729} = 9$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$
$\sqrt[3]{x^3} = x$	$\sqrt[3]{x^6} = x^2$

Exercise (1)

[1] Complete the following table:

Number a	8	125	-27	....	$3\frac{3}{8}$	$-\frac{8}{125}$	....	....
$\sqrt[3]{a}$	....	....	....	-10	....	....	6	-4

[2] Find each of the following:

(1) $\sqrt[3]{216}$ = ...

(2) $\sqrt[3]{-343}$ = ...

(3) $\sqrt[3]{\frac{64}{125}}$ = ………………

(4) $\sqrt[3]{\frac{-8}{27}}$ = ………………

(5) $\sqrt[3]{0.001}$ = ………………

(6) $\sqrt[3]{-2\frac{10}{27}}$ = ………………

(7) $\sqrt[3]{8x^3}$ = ………………

(8) $\sqrt[3]{-27a^6}$ = ………………

[3] Complete:

(1) $\sqrt[3]{x^3}$ = ………………

(2) $\sqrt[3]{\dots\dots\dots}$ = 4

(3) $\sqrt{16} = \sqrt[3]{\dots\dots\dots}$

(4) $\left|\sqrt[3]{-125}\right| = \sqrt{\dots\dots\dots}$

(5) $\sqrt[3]{8} + \sqrt[3]{-8}$ = ………………

(6) $\sqrt[3]{27} - \sqrt[3]{64}$ = ………………

(7) $\sqrt[3]{27} - \sqrt[3]{-27}$ = ………………

(8) $\sqrt{9} + \sqrt[3]{-8}$ = ………………

(9) $\sqrt{64} - \sqrt[3]{64}$ = ………………

(10) $-\sqrt[3]{-1} - \sqrt{1}$ = ………………

(11) $\frac{-\sqrt[3]{64}}{\sqrt{64}}$ = ………………

(12) $\sqrt[3]{64} = \sqrt{\ldots\ldots\ldots}$

(13) $\sqrt[3]{64 + \ldots\ldots} = 5$

[4] Find the value of x in each of the following:

(1) $\sqrt[3]{x} = 5$ $x = \ldots\ldots$

(2) $\sqrt[3]{x} = \frac{-1}{4}$ $x = \ldots\ldots$

(3) $\sqrt[3]{x} = -\sqrt{4}$ $x = \ldots\ldots$

(4) $\sqrt[3]{x} - 3 = -1$ $x = \ldots\ldots$

(5) $x^3 = -8$ $x = \ldots\ldots$

(6) $x^3 = 64$ $x = \ldots\ldots$

(7) $x^3 + 5 = 32$ $x = \ldots\ldots$

(8) $2x^3 = 54$ $x = \ldots\ldots$

(9) $\frac{1}{5}x^3 = -200$ $x = \ldots\ldots$

[5] Choose the correct answer from those given:

(1) $\sqrt[3]{(-8)^2} = \ldots\ldots\ldots$ (2 , -2 , 4 , -4)

(2) $\sqrt[3]{-64} + \sqrt{16} = \ldots\ldots\ldots$ (0 , 8 , -8 , ±8)

(3) $\sqrt{25} - \sqrt[3]{-125} = \ldots\ldots$ (10 , 0 , 5 , ±5)

(4) $\sqrt{(-2)^2} + \sqrt[3]{(-2)^3} = \ldots\ldots\ldots$ (-4 , 8 , 4 , 0)

(5) $\sqrt[3]{3\frac{3}{8}} + \sqrt{0.25} = \ldots\ldots$ ($\frac{3}{2}$, $\frac{1}{2}$, 2 , -2)

(6) $\sqrt[3]{x} = \frac{1}{4}$, then x = …… ($\frac{1}{2}$, $\frac{1}{16}$, $\frac{1}{12}$, $\frac{1}{64}$)

(7) If the volume of a cube is 64 cm^3, then the length of its edge = ……… cm (8 , 4 , 32 , 16)

(8) If the capacity of a cubic vessel is 8 litres, then the length of its inner edge is ……… cm (2 , 4 , 20 , 40)

(9) If the volume of a sphere is 36 π cm^3, then the length of its diameter = ……… cm (3 , 6 , 9 , 27)

(10) If $-\sqrt{25} = \sqrt[3]{y}$, then y = ……… (5 , -5 , 125 , -125)

(11) If x^3 = 64, then $\sqrt{x} = \ldots\ldots\ldots$ (4 , -4 , 2 , -2)

(12) $\sqrt[3]{x^6} = \sqrt{\ldots\ldots}$ (x^3 , x^2 , x , x^4)

(13) If $\frac{x}{3} = \frac{9}{x^2}$, then x = ……… (1 , 3 , 9 , 27)

[6] Find the S.S. of each of the following equations in Q:

(1) $x^3 + 27 = 0$

(2) $8x^3 + 7 = 8$

(3) $(x+3)^3 = 343$

(4) $(5x-2)^3 + 10 = 18$

(5) $2x^3 - 5 = x^3 + 3$

Sheet (3)

The Set of irrational numbers Q`

The set of irrational numbers denoted by Q` appear in:

(1) The square root of a non perfect square of a rational number such as: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ……

(2) The cube root of a non perfect cube of a rational number such as: $\sqrt[3]{2}$, $\sqrt[3]{3}$, $\sqrt[3]{4}$, ……

(3) $\pi \notin Q$

[1] In each of the following, show which of them is a rational number and which of them is an irrational number:

(1) -5

(2) $2\frac{2}{3}$

(3) 2.06

(4) 2.3×10^5

(5) $-\sqrt{36}$

(6) $\sqrt[3]{36}$

(7) Zero

(8) $\sqrt[3]{\frac{-64}{81}}$

(9) $\sqrt{\frac{1}{3}}$

(10) $\frac{\pi}{2}$

(11) $(-5)^{zero}$

(12) $\sqrt{9}+\sqrt{16}$

(13) $\sqrt{4}-\sqrt{11}$

(14) $\sqrt[3]{8}+\sqrt[3]{27}$

[2] Find an approximated value for each of the following numbers:

(1) $\sqrt{11} \cong$ .... (to the nearest hundredth)

(2) $\sqrt[3]{7} \cong$ .... (to the nearest tenth)

(3) $\sqrt[3]{-9} \cong$ .... (to the nearest tenth)

[3] Find two successive integers for each the following numbers to be included between them:

(1) $\sqrt{5}$ is between .... and ....

(2) $\sqrt{12}$ is between .... and ....

(3) $\sqrt[3]{-20}$ is between .... and ....

[4] If x is an integer, find the value of x in each of the following cases:

(1) $x < \sqrt{2} < x+1$ $X =$ ....

(2) $x < \sqrt{80} < x+1$ $X =$ ....

(3) $x < \sqrt[3]{50} < x+1$ $X =$ ....

(4) $x < \sqrt[3]{-100} < x+1$ $X =$ ....

(5) $x < \left| -\sqrt{35} \right| < x+1$ $X =$ ....

[5] Complete the following:

(1) The value of $\sqrt[3]{13}$ to the nearest one decimal is ....

(2) The two consecutive integers which include the number $\sqrt{5}$ between them are .... and ....

(3) $\sqrt[3]{x^6} = \sqrt{\ldots\ldots}$

(4) The solution set in Q` of the equation $5x^2=20$ is ....

(5) If $x \in Z$ and $x < \sqrt[3]{29} < x+1$, then x = ...

(6) If $x=\sqrt{3}$, then $x^2=$ ....

[6] Choose the correct answer:

(1) The irrational number in the following numbers is ....

(a) $\sqrt{\frac{1}{4}}$ (b) $\sqrt[3]{8}$ (c) $\sqrt{\frac{4}{9}}$ (d) $\sqrt{2}$

(2) $(\sqrt[3]{-3})^3$ = ...

(a) 3 (b) -3 (c) ± 3 (d) $\sqrt[3]{-9}$

(3) $\sqrt[3]{9}$ .... $\sqrt{2}$

(a) < (b) > (c) = (d) $\le$

(4) The irrational number located between 2 and 3 is ....

(a) $\sqrt{10}$ (b) $\sqrt{7}$ (c) 2.5 (d) $\sqrt{3}$

(5) The irrational number between 3 and 4 is ....

(a) 3.6 (b) $\sqrt{6}$ (c) $\sqrt{15}$ (d) $\sqrt{17}$

(6) The irrational number between -2 and -1 is ....

(a) -3 (b) $-1\frac{1}{2}$ (c) $-\sqrt{3}$ (d) $\sqrt{2}$

(7) $\sqrt{10} \cong$ ....

(a) 2.99 (b) 3.71 (c) 3 (d) -3.2

(8) The nearest integer to $\sqrt[3]{26}$ is ....

(a) 5 (b) 3 (c) 2 (d) 13

(9) If $n \in Z_+$, $n < \sqrt{26} < n+1$, then $n=$ ......

(a) 25 (b) 5 (c) -5 (d) 24

(10) The area of a square whose side length is $\sqrt{3}$ cm is ……… cm^2.
(a) $4\sqrt{3}$ (b) 9 (c) 3 (d) 6

(11) The square whose side length is $\sqrt{7}$ cm, its area is ……… cm^2.
(a) 28 (b) 49 (c) 7 (d) 14

(12) The square whose area is 10 cm^2, its side length is ……… cm.
(a) 5 (b) -5 (c) $\sqrt{10}$ (d) $-\sqrt{10}$

(13) The S.S. of the equation $(x-\sqrt{5})(x+\sqrt{3})=0$ in Q\` is ………
(a) $\{\sqrt{5}\}$ (b) $\{-\sqrt{3}\}$ (c) $\{-\sqrt{5},\sqrt{3}\}$ (d) $\{\sqrt{5},-\sqrt{3}\}$

(14) $\left|\sqrt[3]{-125}\right| = \sqrt{\ldots\ldots\ldots}$
(a) 25 (b) -25 (c) 5 (d) -5

[7] Find the value of x in each of the following cases and determine whether $x \in Q$ or $x \in Q`$:

(1) $5x^2 = 10$ "$\pm\sqrt{2}$"

(2) $4x^2 = 9$ "$\pm\frac{3}{2}$"

(3) $x^3 = 125$ "5"

(4) $3x^3 = 27$ "$\sqrt[3]{9}$"

(5) $0.001x^3 = -8$ "-20"

(6) $(x-1)^2 = 4$ "$3 \text{ or } -1$"

(7) $(x-5)^3 = 1$ "6"

[8] Find in Q\` the S.S. of each of the following equations:

(1) $x^2 = 13$

(2) $x^3 = 16$

(3) $\frac{2}{5}x^2 = \frac{25}{2}$

(4) $125x^3 - 7 = 20$

(5) $\frac{1}{4}x^2 + 2 = 66$

[9] Prove that (V.I):

(1) $\sqrt{2}$ is included between 1.4 and 1.5

(2) $\sqrt{11}$ is included between 3.31 and 3.32

(3) $\sqrt{5}$ lies between 2.2 and 2.3

(4) $\sqrt[3]{15}$ lies between 2.4 and 2.5

(5) $\sqrt[3]{-17}$ lies between -2.6 and -2.5

(6) $\sqrt{3}+1$ lies between 2.7 and 2.8

Representing an irrational number on the number line

Therefore we can deduce that :

Each irrational number can be represented by a point on the number line.

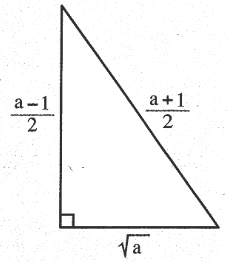
Generally

To draw a line segment with length $\sqrt{a}$ length unit where $a > 1$,

draw a right-angled triangle in which

the length of one side of the right-angle $= \frac{a-1}{2}$ length unit

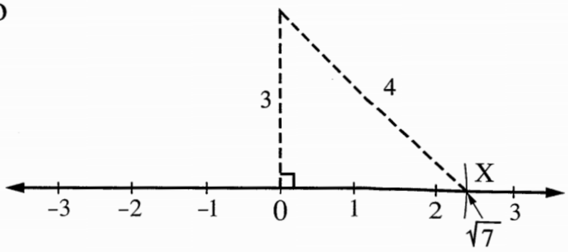
and the length of the hypotenuse $= \frac{a+1}{2}$ length unit.



Example:

Draw a line segment with length $= \sqrt{7}$ length unit , then use it to determine the points which represent the following numbers on the number line :

Using the compasses with a distance equal to the length of $\overline{BC}$ taking O as a centre , draw an arc to cut the number line on the right side of O at the point X , then X is the point which represents $\sqrt{7}$



[10] Determine the point that represents each of the following numbers on the number line:

(1) $\sqrt{3}$

(2) $-\sqrt{11}$

(3) $\sqrt{10}$

(4) $\sqrt{5}+1$

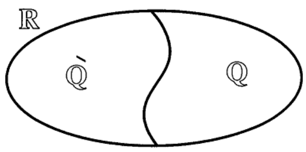
Sheet (4)

The Set of real numbers R

The set of real numbers

It is the set obtained from the union of the set of rational numbers and the set of irrational numbers. It is denoted by $\mathbb{R}$

i.e. $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$ (as shown in the opposite figure)

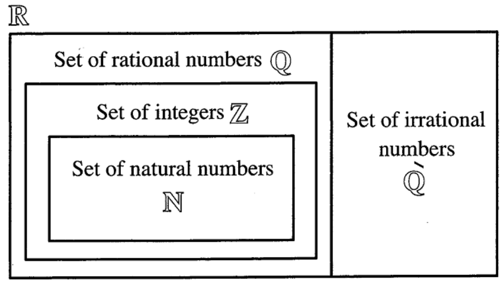


Noticing that : $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$

- **The opposite Venn diagram shows that :**

  $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

  and $\grave{\mathbb{Q}} \subset \mathbb{R}$



R - Q = Q`

R - Q` = Q

$R_+ = \{x : x \in R, x > 0\}$

$R_- = \{x : x \in R, x < 0\}$

$R_+ \cap R_- = \phi$

$R = R_+ \cup \{0\} \cup R_-$

$\pi \in Q`$

$R^* = R - \{0\} = R_+ \cup R_-$

[1] Complete the following:

(1) $Q \cap Q` =$ ....

(2) $Q \cup Q` =$ ....

(3) $R_+ \cap R_- =$ ....

(4) $R_+ \cup R_- =$ ....

(5) $R - Q` =$ ....

(6) $R - Q =$ ....

(7) The solution set in R of the equation $x^2+9=0$ is …………

(8) The cube whose volume is 8 cm^3, then the sum of the lengths of its edges is ………… cm.

(9) The two integers which include the number $\sqrt{12}$ between them are ……… and ………

(10) If $\sqrt[3]{x}=-5$, then $x=$………

[2] Put the suitable sign (<), (>) or (=):

(1) $\sqrt{5}$ ☐ 2

(2) $\sqrt{7}$ ☐ 2.6

(3) $\sqrt[3]{24}$ ☐ 3

(4) $\sqrt[3]{-24}$ ☐ -2

(5) $3-\sqrt{5}$ ☐ $\sqrt[3]{-1}$

(6) $\sqrt[3]{-8}$ ☐ $\sqrt{4}$

(7) $1+\sqrt{3}$ ☐ $\sqrt{5}$

(8) $\sqrt[3]{3}-1$ ☐ 0.2

(9) $\sqrt{2}-1$ ☐ $1-\sqrt{2}$

[3] Choose the correct answer from the given ones:

(1) R = ………
(a) $Q\cup Q`$ (b) $Z_+\cup Z_-$ (c) $R_+\cup R_-$ (d) $N\cup R_-$

(2) $\{x : x\in R,\ x<0\}=$ ………
(a) R_+ (b) R_- (c) R^* (d) R

(3) If x is a negative real number, then which of the following numbers is positive?

(a) x^2 (b) x^3 (c) $2x$ (d) $\frac{x}{2}$

(4) R_+ = ...

(a) $\{x : x \in R,\ x < 0\}$ (b) $\{x : x \in R,\ x \geq 1\}$

(c) $\{x : x \in R,\ x > 0\}$ (d) $\{x : x \in R,\ x \geq 0\}$

(5) $\sqrt[3]{5}$ ...... $\sqrt{3}$

(a) < (b) > (c) = (d) $\geq$

(6) The irrational number which is included between 2 and 3 is ....

(a) $\sqrt{10}$ (b) $\sqrt{7}$ (c) 2.5 (d) $\sqrt{3}$

(7) $(-5)^{zero}$ = ...

(a) zero (b) 1 (c) -1 (d) -5

(8) The S.S. of the equation $x^2 + 1 = 0$ in R is ....

(a) $\{-1\}$ (b) $\{1, -1\}$ (c) $\{1\}$ (d) ϕ

(9) $\sqrt{(2-\pi)^2}$ .... $(2-\pi)$

(a) < (b) > (c) = (d) $\geq$

(10) If $x \in R_+$, $y \in R_+$ and $x^2 > y^2$, then ....

(a) x > y (b) x < y (c) x = y (d) $x \leq y$

(11) If $\frac{1}{a}$ and $\frac{a}{\sqrt{5}}$ are real numbers included between 0 and 1, then a = ...

(a) -2 (b) 1 (c) $\sqrt{5}$ (d) 2

[4] Arrange the following numbers in an ascending order:

(1) $\sqrt{8}$, $-\sqrt{3}$, $\sqrt{15}$, $\sqrt{5}$, $-\sqrt{7}$ and $-\sqrt{11}$

The order is: ....

(2) $\sqrt{27}$, $-\sqrt{45}$, $\sqrt{20}$, 0.6 and $\sqrt[3]{-1}$

The order is: ……………………………………………………………………

[5] Arrange the following numbers in a descending order:

(1) $\sqrt{62}$, 8 , $-\sqrt{50}$ and $\sqrt{70}$

The order is: ……………………………………………………………………

(2) $\sqrt{6}$, 9 , $-\sqrt{10}$, $-\sqrt{7}$, $-\sqrt{50}$ and $\sqrt{101}$

The order is: ……………………………………………………………………

[6] Write three positive irrational numbers less than 2:

……………………………………………………………………

[7] Write three negative irrational numbers greater than $-\sqrt{6}$:

……………………………………………………………………

[8] Write four irrational numbers included between 15 and 17:

……………………………………………………………………

[9] Write three irrational positive numbers less than 3:

……………………………………………………………………

[10] Solve the following equations to the nearest hundredth given $x \in R$:

(1) $x^2 - 6 = 0$

(2) $\frac{3}{4}x^2 = 24$

(3) $\frac{1}{2}x^2 - 5 = 0$

(4) $5x^3 + 3 = 2$

(5) $(x^2 - 9)(x^3 - 5) = 0$

(6) $(2x^3 - 5)(x^2 + 1) = 0$

Rules to solve geometric applications

The cube

Let the edge length = $\mathbb{L}$,

volume = $\mathbb{L}^3$, lateral area = $4\mathbb{L}^2$, total area = $6\mathbb{L}^2$

[11] Find the edge length of a cube whose volume is 1.728 cm^3. Is the edge length a rational number? "$\frac{6}{5}cm$"

[12] A cube whose total area is 13.5 cm^2. Find its edge length. Is the edge length a rational number? "*1.5 cm*"

The Square

Let the edge length = S, then $area = S^2$, $S = \sqrt{area}$

Let the diagonal length = d, then $area = \frac{1}{2}d^2$, $d = \sqrt{2 \times area}$ or $d = \sqrt{2 \times s \times s}$

[13] Find the side length of a square whose area is 5 cm^2. Is the edge length a rational number? "$\sqrt{5}\,cm$"

....

....

....

[14] A square is of area 32 cm^2, Find its side length and its diagonal length? "$\sqrt{32}, 8$"

....

....

....

[15] A square is of side length 6 cm. Find its diagonal length?"$\sqrt{72}\,cm$"

....

....

....

Sheet (5)

Subsets of real numbers (Intervals)

Sheet (5) Subsets of real numbers (Intervals)

If X is the set of integers which greater than or equal to -3 and less than 2, we can express the set X by the description method as follows: $X = \{a : a \in Z, -3 \le a < 2\}$ we can express it by listing method $X = \{-3, -2, -1, 0, 1, 2\}$
If $K = \{a : a \in R, -3 \le a < 2\}$ but it is impossible to express the set K by listing method because there are an infinity of real numbers between -3 and 2. So we use another method to express a subset of the set of real numbers, which is (the intervals).

Types of intervals

Types of intervals		The interval	Expression by distinguished property	Representation on the number line	Notice that
The limited intervals	Closed	$[a , b]$	$\{x : x \in \mathbb{R} , a \le x \le b\}$	a b	• $a \in [a , b]$ • $b \in [a , b]$
	Opened	$]a , b[$	$\{x : x \in \mathbb{R} , a < x < b\}$	a b	• $a \notin]a , b[$ • $b \notin]a , b[$
	half opened (half closed)	$[a , b[$	$\{x : x \in \mathbb{R} , a \le x < b\}$	a b	• $a \in [a , b[$ • $b \notin [a , b[$
		$]a , b]$	$\{x : x \in \mathbb{R} , a < x \le b\}$	a b	• $a \notin]a , b]$ • $b \in]a , b]$
The unlimited intervals		$[a , \infty[$	$\{x : x \in \mathbb{R} , x \ge a\}$	a	$a \in [a , \infty[$
		$]a , \infty[$	$\{x : x \in \mathbb{R} , x > a\}$	a	$a \notin]a , \infty[$
		$]-\infty , a]$	$\{x : x \in \mathbb{R} , x \le a\}$	a	$a \in]-\infty , a]$
		$]-\infty , a[$	$\{x : x \in \mathbb{R} , x < a\}$	a	$a \notin]-\infty , a[$

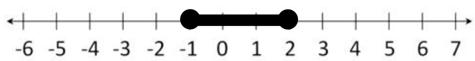
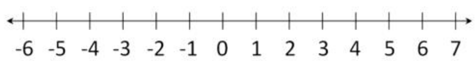
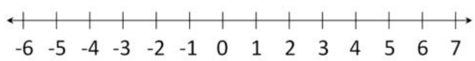
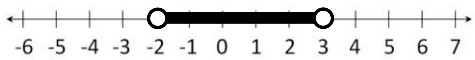
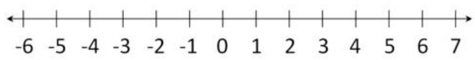
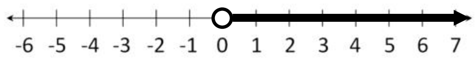
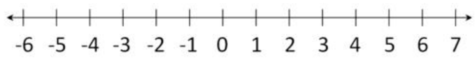
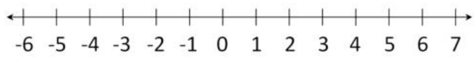
Remarks

1 $\mathbb{R} =]-\infty , \infty[$ **2** $\mathbb{R}_+ =]0 , \infty[$ **3** $\mathbb{R}_- =]-\infty , 0[$

4 The set of non-negative real numbers $= \mathbb{R}_+ \cup \{0\} = [0 , \infty[$

5 The set of non-positive real numbers $= \mathbb{R}_- \cup \{0\} =]-\infty , 0]$

[1] Complete the following table:

(1)	[-1 , 2]	$\{x : x \in R, -1 \le x \le 2\}$	
(2)	[1 , 3[	....	
(3)	....	$\{x : x \in R, 0 < x \le 3\}$	
(4)	....	....	
(5)	]-∞ , 1]	....	
(6)	....	....	
(7)	....	$\{x : x \in R, x < 4\}$	
(8)	[-2 , ∞[	....	

[2] Complete using (∈) or (∉):

(1) 3 .... [3 , 5]

(2) -2 ....]-2 , 1]

(3) 0 .... [-1 , 4[

(4) |-3| .... [2 , ∞[

(5) $\sqrt{9}$ ....]-3 , ∞[

(6) $\sqrt[3]{-1}$ ....]-∞ , 1]

(7) 1.3×10^{-5} .... R_+

(8) $\sqrt{2}$ .... [2 , 5]

(9) 5 ....]$\sqrt{5}$, $\sqrt{23}$ [

(10) $\sqrt[3]{-125}$ ......]$-\sqrt{25}$, $\sqrt{25}$]

[3] Choose the correct answer:

1	$\sqrt[3]{8}$ .... $]-\infty , 4[$ (a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$
2	$5 \in$ .... (a) $]5 , \infty[$ (b) $]-\infty , 5[$ (c) $(3 , 5)$ (d) $[-5 , \infty[$
3	ose the correct answer : The opposite figure represents the interval ....  (a) $[-4 , 8[$ (b) $[8 , -4]$ (c) $[-4 , 8]$ (d) $]-4 , 8[$
4	$\mathbb{R} =$ .... (a) $\mathbb{R}_+ \cap \mathbb{R}_-$ (b) $\mathbb{R}_+ \cup \mathbb{R}_-$ (c) $]-\infty , \infty[$ (d) $\mathbb{Q} \cap \grave{\mathbb{Q}}$
5	$\mathbb{R}_+ =$ .... (a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$
6	$\mathbb{R}_- =$ .... (a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$
7	The set of non-negative real numbers = ... (a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$
8	The set of non-positive real numbers = ... (a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$

Operations on Intervals

[1] Complete:

1	$[1, 3] \cup [2, 5[=$ ……………
2	$]1, 3] \cup [2, 5] =$ ……………
3	$]-\infty, 1] \cup [-4, \infty[=$ ……….
4	$]3, 5[\cup \{3, 5\} =$ ……………
5	$]-2, 2] \cup \{-2, 0\} =$ ……………
6	$]5, 7[\cup \{5, 7\} =$ ……………
7	$\mathbb{N} \cap]1, 2[=$ ……………
8	$]1, 7[\cap]3, 5[=$ ……………
9	$[-3, 1[\cap [-1, 4[=$ ……………
10	$[-2, 5] \cap]4, 6] =$ ……….
11	$]-3, 5] \cap [0, 3[=$ ……………
12	$[1, 5] - \{1, 5\} =$ ……………
13	$[2, 5] - \{5\} =$ ……….
14	$[2, 5] - \{2, 5\} =$ ……….
15	$[3, 4] - \{3, 4\} =$ ……………

16	$[3, 5] - \{3\} =$ ……………
17	$[3, 7[-]-2, 5] =$ ……………
18	$[-4, 6] -]-4, 6[=$ ……………
19	$[-1, 5] -]-1, 5[=$ ………
20	$[2, 7] -]2, 7[=$ ……………

[2] Essay problems:

1	If $X = [-1, 4]$, $Y = [3, \infty[$, $Z = \{3, 4\}$, **find using the number line :** (1) $X \cup Y$ (2) $X \cap Y$ (3) $X - Z$ ………………………………………………………………………………………………
2	If $X = [3, \infty[$, $Y =]-4, 8[$ **Find :** (1) $X \cup Y$ (2) $X \cap Y$ (3) $\grave{X}$ ………………………………………………………………………………………………
3	**Find each of the following :** (1) $[0, 5] \cup [3, 8[$ (2) $[1, 5] \cap]-2, 3]$ ………………………………………………………………………………………………
4	If $X = [-2, 3]$, $Y = [1, 5[$, **then find by using the number line :** $X \cup Y$, $X - Y$ ……………………………………………………………………………………………… ………………………………………………………………………………………………

Homework

[1] Choose the correct answer:

1	$]-1, 3] \cap [-3, -1] = \dots\dots$ (a) $\varnothing$ (b) $\{-3\}$ (c) $\{-1\}$ (d) $\{3\}$
2	$[1, 5] \cap]-2, 3] = \dots\dots$ (a) $\{1, 3\}$ (b) $]1, 3[$ (c) $[1, 3]$ (d) $[1, 3[$
3	$]-3, 5[\cap [0, 3[= \dots\dots$ (a) $[0, 3]$ (b) $[0, 3[$ (c) $]-3, 0[$ (d) $[3, 5[$
4	$[2, 7] - \{2, 7\} = \dots\dots$ (a) $[1, 6]$ (b) $\varnothing$ (c) $]2, 7[$ (d) $\{0\}$
5	$[-2, 5] - \{-2, 6\} = \dots\dots$ (a) $]-2, 5[$ (b) $]-2, 6[$ (c) $]-2, 5]$ (d) $[-2, 5[$
6	$[-3, 7] - \{-3, 7\} = \dots\dots$ (a) $[-3, 7[$ (b) $]-3, 7]$ (c) $]-3, 7[$ (d) $(0, 0)$

[2] Essay problems:

1	If $X = [-1, 4]$, $Y = [3, \infty[$, $Z = \{3, 4\}$, **find using the number line :** (1) $X \cup Y$ (2) $X \cap Y$ (3) $X - Z$ ....
2	If $X = [-2, 1]$ and $Y = [0, \infty[$ **Find :** (1) $X \cap Y$ (2) $X \cup Y$ ....

3	If $X = [3 , \infty[$, $Y =]-4 , 8[$ **Find :** (1) $X \cup Y$ (2) $X \cap Y$ (3) $\grave{X}$ ....
4	If $X = [-1 , 4]$ and $Y = [2 , 7]$ **, then find each of :** (1) $X \cap Y$ (2) $Y \cup X$ ....
5	If $X = [-2 , 1]$, $Y = [0 , \infty[$ **Find :** (1) $X \cap Y$ (2) $X \cup Y$ (3) $Y - X$ ....
6	If $X = [-1 , 4]$, $Y = [3 , \infty[$ **, find using the number line each of :** (1) $X \cup Y$ (2) $X - Y$ ....
7	**Find each of the following :** (1) $[0 , 5] \cup [3 , 8[$ (2) $[1 , 5] \cap]-2 , 3]$ ....
8	If $X = [-2 , 3]$, $Y = [1 , 5[$ **, then find by using the number line :** $X \cup Y$, $X - Y$ ....

9	If $X = [-2 , 4]$ and $Y =]2 , \infty[$, **find each of the following using the number line :** (1) $X \cap Y$ (2) $X - Y$ ....
10	If $X = [-1 , 4]$, $Y = [2 , 7]$, then find each of the following by using the number line : (1) $X \cup Y$ (2) $X \cap Y$ (3) $X - Y$ ....
11	If $X = [1 , 5]$, $Y = [2 , 7]$ **Find by using the number line :** (1) $X \cap Y$ (2) $X \cup Y$ ....

Sheet (6)

Operations on the real numbers

We know that 2x and 3x are two like algebraic terms, then 2x + 3x = 5x then we deduce that $2\sqrt{5}+3\sqrt{5}=(2+3)\sqrt{5}=5\sqrt{5}$, but 2x and 3y are two unlike algebraic terms then their sum 2x + 3y then the sum of $2\sqrt{3}, 3\sqrt{2}$ written in the form $2\sqrt{3}+3\sqrt{2}$

Properties of addition of real numbers

Closure :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ we find that $(a + b) \in \mathbb{R}$

i.e. The sum of any two real numbers is a real number, therefore we say $\mathbb{R}$ is closed under addition operation.

For example :

- $\sqrt{5} \in \mathbb{R}$ and $2\sqrt{5} \in \mathbb{R}$ we find that : $\sqrt{5}+2\sqrt{5}=3\sqrt{5} \in \mathbb{R}$

Commutative property :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a + b = b + a$

For example :

$5\sqrt[3]{2}+4\sqrt[3]{2}=9\sqrt[3]{2}$, $4\sqrt[3]{2}+5\sqrt[3]{2}=9\sqrt[3]{2}$

i.e. $5\sqrt[3]{2}+4\sqrt[3]{2}=4\sqrt[3]{2}+5\sqrt[3]{2}$

Associative property :

For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a + b) + c = a + (b + c) = a + b + c$

For example :

$(\sqrt{3}+2\sqrt{3})+5\sqrt{3}=3\sqrt{3}+5\sqrt{3}=8\sqrt{3}$,

$\sqrt{3}+(2\sqrt{3}+5\sqrt{3})=\sqrt{3}+7\sqrt{3}=8\sqrt{3}$

i.e. $(\sqrt{3}+2\sqrt{3})+5\sqrt{3}=\sqrt{3}+(2\sqrt{3}+5\sqrt{3})$

The additive neutral :

For every $a \in \mathbb{R}$ it will be $a + 0 = 0 + a = a$

i.e. Zero is the additive neutral.

For example : $\sqrt{2}+0=0+\sqrt{2}=\sqrt{2}$, $-\sqrt[3]{5}+0=0+(-\sqrt[3]{5})=-\sqrt[3]{5}$

The additive inverse of every real number :

For every $a \in \mathbb{R}$ there is $(-a) \in \mathbb{R}$ where $a + (-a) =$ zero (the additive neutral)

For example :

- The additive inverse of the number $\sqrt{3}$ is $-\sqrt{3}$ and vice versa because $\sqrt{3} + (-\sqrt{3}) = 0$
- The additive inverse of the number $2 + \sqrt{5}$ is $-(2 + \sqrt{5})$ and equals $-2 - \sqrt{5}$
- The additive inverse of the number $3 - \sqrt{2}$ is $-(3 - \sqrt{2})$ and equals $\sqrt{2} - 3$
- The additive inverse of the number zero is itself.

[1] Find the result of each of the following in the simplest form:

(1)	$\sqrt{3} + 2\sqrt{3}$
(2)	$3\sqrt{2} - 5\sqrt{2}$
(3)	$2\sqrt{5} - 3\sqrt{5} + \sqrt{5}$
(4)	$5\sqrt[3]{7} - 8\sqrt[3]{7} + 2\sqrt[3]{7}$
(5)	$4\sqrt{5} - 2\sqrt{5} + 5\sqrt{5} - \sqrt{5}$
(6)	$5\sqrt{3} - 7\sqrt{3} + 3\sqrt{3} - \sqrt{3}$
(7)	$\sqrt{5} - \sqrt{3} + 2\sqrt{5} + \sqrt{3}$
(8)	$2\sqrt{3} + 5 + \sqrt{3} - 6$
(9)	$2\sqrt{7} - 3\sqrt{2} + \sqrt{7} + 5\sqrt{7}$
(10)	$2\sqrt{2} - 3\sqrt[3]{2} + 5\sqrt{2} + \sqrt[3]{2}$
(11)	$8\sqrt{\frac{1}{4}} + 2\sqrt[3]{3} - \sqrt[3]{64} - 5\sqrt[3]{3}$
(12)	$\frac{1}{4}\sqrt{2} + \frac{2}{7}\sqrt{5} + \frac{3}{4}\sqrt{2} - \frac{2}{7}\sqrt{5}$

The properties of multiplication operation of real numbers

Closure :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b \in \mathbb{R}$

i.e. The product of any two real numbers is a real number therefore we say :

The multiplication operation is closed in $\mathbb{R}$

For example :

- $\sqrt{3} \in \mathbb{R}$ and $2\sqrt{3} \in \mathbb{R}$

  We find that : $\sqrt{3} \times 2\sqrt{3} = 2 \times 3 = 6 \in \mathbb{R}$

Commutative property :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b = b \times a$

For example :

- $2\sqrt{5} \times 3\sqrt{5} = 6 \times 5 = 30$, $3\sqrt{5} \times 2\sqrt{5} = 6 \times 5 = 30$

  i.e. $2\sqrt{5} \times 3\sqrt{5} = 3\sqrt{5} \times 2\sqrt{5}$

The associative property :

For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a \times b) \times c = a \times (b \times c) = a \times b \times c$

For example :

- $(2\sqrt{7} \times 4\sqrt{7}) \times \sqrt{7} = 56 \times \sqrt{7} = 56\sqrt{7}$
- $2\sqrt{7} \times (4\sqrt{7} \times \sqrt{7}) = 2\sqrt{7} \times 28 = 56\sqrt{7}$

  i.e. $(2\sqrt{7} \times 4\sqrt{7}) \times \sqrt{7} = 2\sqrt{7} \times (4\sqrt{7} \times \sqrt{7})$

The multiplicative neutral :

For every $a \in \mathbb{R}$ it will be $a \times 1 = 1 \times a = a$

i.e. One is the multiplicative neutral in $\mathbb{R}$

For example :

- $\sqrt[3]{5} \times 1 = 1 \times \sqrt[3]{5} = \sqrt[3]{5}$

The multiplicative inverse of any non-zero real number :

For every real number $a \neq 0$, there is a real number $\frac{1}{a}$ where $a \times \frac{1}{a} = 1$ which is the multiplicative neutral.

For example :

- The multiplicative inverse of $\sqrt{3}$ is $\frac{1}{\sqrt{3}}$ because $\sqrt{3} \times \frac{1}{\sqrt{3}} = 1$
- The multiplicative inverse of $-\frac{\sqrt{2}}{5}$ is $-\frac{5}{\sqrt{2}}$
- The multiplicative inverse of the number 1 is itself and also the multiplicative inverse of -1 is itself.

Notice that :

Both the number and its multiplicative inverse have the same sign.

Notice that :

There is no multiplicative inverse for the number zero because $\frac{1}{\text{zero}}$ is meaningless (**i.e.** undefined)

Remark

- Since each non-zero real number has a multiplicative inverse then the division operation by any real number does not equal zero is possible in $\mathbb{R}$ and it is defined as

  For every $a \in \mathbb{R}$ and $b \in \mathbb{R}^*$ it will be $a \div b = a \times \frac{1}{b}$

  i.e. The division operation $(a \div b)$ means multiplying the number a by the multiplicative inverse of the number b such that $b \neq 0$

Then we can deduce that :

The division operation in $\mathbb{R}$ is not commutative and it is not associative.

[2] Find the result of each of the following in the simplest form:

(1)	$\sqrt{3} \times \sqrt{3}$
(2)	$-2\sqrt{5} \times 3\sqrt{5}$
(3)	$2 \times 3\sqrt{2}$
(4)	$\frac{1}{3}\sqrt{3} \times \sqrt{3}$
(5)	$(\sqrt[3]{5})^3 \times 3\sqrt{3}$
(6)	$2\sqrt{3} \times \frac{2\sqrt{7}}{7} \div \frac{20\sqrt{3}}{5\sqrt{7}}$

[3] Make the denominator in each of the following an integer:

(1)	$\frac{3}{\sqrt{3}}$
(2)	$\frac{10}{\sqrt{5}}$
(3)	$\frac{-6}{\sqrt{3}}$
(4)	$\frac{6}{2\sqrt{3}}$
(5)	$\frac{\sqrt{2}+3}{\sqrt{2}}$

Distributing multiplication on addition and subtraction

For any three real numbers a , b and c it will be :

- $a(b \pm c) = ab \pm ac$
- $(b \pm c)a = ba \pm ca$

Remarks:

- $(a+b)(a-b) = a^2 - b^2$
- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$

[4] Find the result of each of the following in the simplest form:

(1)	$2(\sqrt{2}+\sqrt{5})$
(2)	$\sqrt{2}(5+\sqrt{2})$
(3)	$\sqrt{7}(\sqrt{7}+2)$
(4)	$-\sqrt{3}(-5-\sqrt{3})$
(5)	$-2\sqrt{5}(3-\sqrt{5})$
(6)	$\sqrt{7}\left(\frac{2}{\sqrt{7}}-\sqrt{7}+3\right)$

[5] Find the result of each of the following operations:

(1)	$(\sqrt{2}+1)(\sqrt{2}-1)$
(2)	$(4-3\sqrt{2})(4+3\sqrt{2})$
(3)	$(\sqrt{5}-1)^2$
(4)	$(2\sqrt{3}+4)^2$

[6] Complete the following:

(1)	The multiplicative neutral in $\mathbb{R}$ is ………. and the additive neutral in $\mathbb{R}$ is ……….
(2)	The additive inverse of the number $1-\sqrt{2}$ is ……….
(3)	The multiplicative inverse of the number $\frac{2\sqrt{3}}{5}$ is $\frac{\dots\dots\dots}{6}$
(4)	The multiplicative inverse of the number $\frac{3}{\sqrt{3}}$ is $\frac{\dots\dots\dots}{\sqrt{3}}$
(5)	If : $a=\sqrt{5}$ and $b=2\sqrt{5}$, then : $ab=$ ……….
(6)	If : $x=\sqrt{5}+2$ and $y=\sqrt{5}-2$ then $(x+y)^2=$ ……….
(7)	If : $x=2\sqrt[3]{5}$, then $x^3=$ ……….
(8)	The solution set of the equation : $x^2+25=0$ in $\mathbb{R}$ is ……….
(9)	$\mathbb{R}_+ \cup [-3\ ,2[=$ ……….

[7] Choose the correct answer:

(1) $2\sqrt{5}+3\sqrt{5}=$ ……….

(a) $5\sqrt{10}$ (b) $5\sqrt{5}$ (c) $6\sqrt{5}$ (d) 30

(2)	The multiplicative inverse of the number $\frac{\sqrt{3}}{6}$ is ……… (a) $\frac{\sqrt{6}}{3}$ (b) $2\sqrt{3}$ (c) $\frac{3}{\sqrt{6}}$ (d) $-\frac{\sqrt{3}}{6}$
(3)	$\sqrt{3}+(-\sqrt{3})=$ ……… (a) $2\sqrt{3}$ (b) $2\sqrt{6}$ (c) $\sqrt{6}$ (d) zero
(4)	$-2\sqrt{3}\times\sqrt{3}=$ ……… (a) -6 (b) $-2\sqrt{3}$ (c) $2\sqrt{3}$ (d) 6
(5)	The additive inverse of the number $\frac{6}{\sqrt{2}}=$ ……… (a) $-2\sqrt{3}$ (b) $2\sqrt{3}$ (c) $-3\sqrt{2}$ (d) $3\sqrt{2}$
(6)	The additive inverse of the number $(\sqrt{2}-\sqrt{5})=$ ……… (a) $\sqrt{2}+\sqrt{5}$ (b) $\sqrt{5}-\sqrt{2}$ (c) $\sqrt{2}-\sqrt{5}$ (d) $-\sqrt{2}-\sqrt{5}$
(7)	The multiplicative inverse of the number $\sqrt{5}$ is ……… (a) -5 (b) $\frac{-1}{5}$ (c) $\frac{5}{\sqrt{5}}$ (d) $\frac{\sqrt{5}}{5}$
(8)	$(\sqrt{5}+3\sqrt{5})\div\sqrt{5}=$ ……… (a) $3\sqrt{5}$ (b) 3 (c) 5 (d) 4
(9)	If : $x=\sqrt{2}+10$, $y=\sqrt{2}-10$, then $(x+y)^2=$ ……… (a) 4 (b) 6 (c) 8 (d) $4\sqrt{2}$
(10)	$[2,5]-\{2,5\}=$ ……… (a) $[3,4]$ (b) $]2,5[$ (c) $\{2,5\}$ (d) $[2,5]$
(11)	If : $x^3+9=1$ where $x\in\mathbb{R}$, then $x=$ ……… (a) -8 (b) -2 (c) 2 (d) 8
(12)	If : $x=\sqrt{3}+2$, then $x^2=$ ……… (a) 5 (b) 7 (c) $7+2\sqrt{3}$ (d) $7+4\sqrt{3}$
(13)	If : $x^2-y^2=60$, $x+y=5\sqrt{6}$, then $=x-y=$ ……… (a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) $3\sqrt{6}$ (d) $4\sqrt{6}$

[8] If $x=\sqrt{5}-2$ and $y=\sqrt{5}+2$, find the value of:

(1) $x+y$

(2) $x-y$

(3) xy

[9] If $x=-\sqrt{3}$ and $y=2\sqrt{3}-3$, find the value of:

(1) $x+y$

(2) xy

(3) $\frac{y}{x}$

Sheet (7)

Operations on the square roots

If a and b are two non negative real numbers , then

1 $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

For example :

- $\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$
- $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

2 $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ where $b \neq 0$

For example :

- $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$
- $\sqrt{\frac{16}{49}} = \frac{\sqrt{16}}{\sqrt{49}} = \frac{4}{7}$

3 $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$ where $b \neq 0$

This operation is carried out to make the denominator an integer.

For example :

- $\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$
- $\sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

[1] Put each of the following in the form $a\sqrt{b}$ where a and b are two integers and b is the least possible value:

(1) $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$

(2) $\sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$

(3) $2\sqrt{27} = 2 \times \sqrt{36 \times 2} = 2 \times 6\sqrt{2} = 12\sqrt{2}$

(4) $\frac{2}{5}\sqrt{1000} = \frac{2}{5}\sqrt{100 \times 10} = \frac{2}{5} \times 10\sqrt{10} = 4\sqrt{10}$

(5) $2\sqrt{\frac{1}{2}} = \sqrt{2^2 \times \frac{1}{2}} = \sqrt{2}$ $\left(x\sqrt{\frac{1}{x}} = \sqrt{x} \right)$

(6) $6\sqrt{\frac{2}{3}} = \sqrt{36 \times \frac{2}{3}} = 2\sqrt{6}$

[2] Simplify each of the following to the simplest form:

(1) $\sqrt{50} + \sqrt{8}$

....
....
....

(2) $3\sqrt{2} + \sqrt{8} - \sqrt{18}$

....
....
....

(3) $\sqrt{98} - \sqrt{128} - \sqrt{18} + 4\sqrt{2}$

....
....
....

(4) $\sqrt{27} + 5\sqrt{18} - \sqrt{300}$

....
....
....

(5) $2\sqrt{18} + \sqrt{50} + \frac{1}{3}\sqrt{162}$

....
....
....

(6) $2\sqrt{5} + 4\sqrt{20} - 5\sqrt{\frac{1}{5}}$

....
....
....

(7) $2\sqrt{5} + 6\sqrt{\frac{1}{3}} - \sqrt{12} - 5\sqrt{\frac{1}{5}}$

....
....
....

(8) $\sqrt{18} - \frac{\sqrt{12}}{\sqrt{6}}$

....
....
....

(9) $2\sqrt{3}\times5\sqrt{2}$

....

....

....

(10) $\sqrt{5}\times2\sqrt{10}$

....

....

....

(11) $\sqrt{\frac{2}{7}}\times\sqrt{\frac{7}{2}}$

....

....

....

(12) $\frac{3\sqrt{15}}{\sqrt{5}}$

....

....

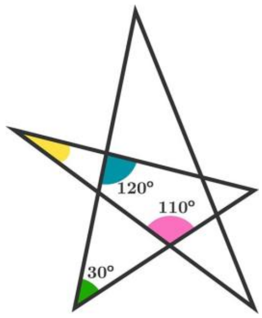
....

(13) $12\sqrt{\frac{2}{3}}\times\sqrt{54}$

....

....

....



[3] Simplify each of the following to the simplest form:

(1)	$\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)$ = ...
(2)	$\left(3\sqrt{5}-\sqrt{7}\right)\left(3\sqrt{5}+\sqrt{7}\right)$ = ...
(3)	$\left(3\sqrt{2}-5\right)\left(3\sqrt{2}+5\right)$ = ...
(4)	$\left(\sqrt{2}+\sqrt{6}\right)^2$ = ...
(5)	$\left(\sqrt{3}-\sqrt{2}\right)^2$ = ...

[4] Choose the correct answer:

(1)	$\frac{\sqrt{63}}{\sqrt{7}} = \dots\dots\dots$ (a) 3 (b) $\sqrt{3}$ (c) 9 (d) ± 3
(2)	🕮 $\sqrt{8} - \sqrt{2} = \dots\dots\dots$ (a) $\sqrt{6}$ (b) $\sqrt{2}$ (c) 2 (d) 1
(3)	🕮 $(\sqrt{8} + \sqrt{2})^2 = \dots\dots\dots$ (a) $\sqrt{10}$ (b) 10 (c) 18 (d) $\sqrt{18}$
(4)	🕮 $(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5}) = \dots\dots\dots$ (a) 2 (b) 12 (c) $2\sqrt{7}$ (d) $-2\sqrt{5}$
(5)	$\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}} = \dots\dots\dots$ (a) 1 (b) $\sqrt{\frac{1}{4}}$ (c) $\sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$
(6)	$\frac{\sqrt{27}}{\sqrt{3}} \div \frac{\sqrt{72}}{\sqrt{2}} = \dots\dots\dots$ (a) $\frac{1}{2}$ (b) 2 (c) -2 (d) 4
(7)	The multiplicative inverse of the number $\sqrt{50}$ is $\dots\dots\dots$ (a) $\frac{\sqrt{2}}{10}$ (b) $\frac{-\sqrt{2}}{10}$ (c) $-5\sqrt{2}$ (d) $5\sqrt{2}$
(8)	If : $x = \frac{\sqrt{6}}{\sqrt{2}}$, then $x^{-1} = \dots\dots\dots$ (a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{3}$ (d) $2\sqrt{3}$

(9) If : $x = \sqrt{7} + \sqrt{3}$ and $y = \sqrt{28} + \sqrt{12}$, then $x =$ ....

(a) y (b) $\frac{1}{2}$ y (c) 2y (d) y^2

[5] If $x = \frac{\sqrt{50} - \sqrt{18}}{2}$ and $y = 2 - \sqrt{2}$, find in the simplest form:

(1) $x + y$

....

(2) xy

....

Sheet (8)

The two conjugate numbers

If a and b are two positive rational numbers , then each of the two numbers $(\sqrt{a}+\sqrt{b})$ and $(\sqrt{a}-\sqrt{b})$ is conjugate to the other one and we find that

- Their sum $= (\sqrt{a}+\sqrt{b}) + (\sqrt{a}-\sqrt{b}) = 2\sqrt{a}$ = twice the first term.
- Their product $= (\sqrt{a}+\sqrt{b})\ (\sqrt{a}-\sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$

- **The difference:**

Greater – smaller = $(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b}) = 2\sqrt{b}$ **(twice the second term)**

Smaller – greater = $(\sqrt{a}-\sqrt{b})-(\sqrt{a}+\sqrt{b}) = -2\sqrt{b}$ **(negative twice the second term)**

Example: $(\sqrt{2}+\sqrt{5})$ its conjugate is $(\sqrt{2}-\sqrt{5})$

Their sum 2 of 1^{st} term	G - S 2 of 2^{nd} term	S - G -2 of 2^{nd} term	Their product $(1^{st})^2 - (2^{nd})^2$
$2\sqrt{2}$	$2\sqrt{5}$	$-2\sqrt{5}$	$2-5=-3$

Exercise (1): $(3-\sqrt{7})$ its conjugate is ....

Their sum	G - S	S - G	Their product

Exercise (2): $(3\sqrt{5}-\sqrt{6})$ its conjugate is ....

Their sum	G - S	S - G	Their product

Remarks:

- $x^2 - y^2 = (x+y)(x-y)$
- $x^2 + 2xy + y^2 = (x+y)^2$
- $x^2 - 2xy + y^2 = (x-y)^2$

[1] Choose the correct answer:

(1)	The conjugate of $(\sqrt{3}-\sqrt{5})$ is ………… (a) $\sqrt{5}-3$ (b) $\sqrt{3}+\sqrt{5}$ (c) $-\sqrt{3}-\sqrt{5}$ (d) $\sqrt{5}-\sqrt{3}$
(2)	$(\sqrt{5}+\sqrt{3})^2(\sqrt{5}-\sqrt{3})^2 =$ ……… (a) 4 (b) 2 (c) 8 (d) 3
(3)	The number $\frac{4}{3+\sqrt{5}}$ in the simplest form is …………… (a) $3+\sqrt{5}$ (b) $3-\sqrt{5}$ (c) $\sqrt{3}+\sqrt{5}$ (d) $3\sqrt{5}$
(4)	[2 , 5] – {2} = …………… (a)]2 , 5[(b)]2 , 5] (c) [2 , 5[(d) [2 , 5]
(5)	The conjugate of the number $\frac{1}{\sqrt{3}+\sqrt{2}}$ is ………… (a) $\sqrt{3}-\sqrt{2}$ (b) $\sqrt{3}+\sqrt{2}$ (c) $\frac{1}{\sqrt{3}-\sqrt{2}}$ (d) $-\sqrt{3}-\sqrt{2}$

[2] If $x=\sqrt{5}+\sqrt{3}$ and $y=\frac{2}{\sqrt{5}+\sqrt{3}}$, prove that x and y are conjugate numbers

then find the value of $x^2+2xy+y^2$.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………

[3] If $x=\sqrt{5}-\sqrt{2}$ and $y=\frac{3}{\sqrt{5}-\sqrt{2}}$, prove that x and y are conjugate numbers

then find the value of $x^2-2xy+y^2$.

....

....

....

[4] If $a=\sqrt{3}+\sqrt{2}$ and $b=\frac{1}{\sqrt{3}+\sqrt{2}}$, find the value of a^2-b^2.

....

....

....

[5] If $x=\frac{4}{\sqrt{7}-\sqrt{3}}$ and $y=\frac{4}{\sqrt{7}+\sqrt{3}}$, find the value of x^2y^2.

....

....

....

[6] If $x=\sqrt{5}+\sqrt{2}$ and $y=\sqrt{5}-\sqrt{2}$, find the value of $\frac{x+y}{xy-1}$.

....

....

....

[7] If $x=\frac{2}{\sqrt{5}-\sqrt{3}}$ and $y=\frac{2}{\sqrt{5}+\sqrt{3}}$, find the value of x^2-xy+y^2.

....

....

....

[8] If $x=\frac{5\sqrt{2}+3\sqrt{5}}{\sqrt{5}}$ and $y=\frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}}$, find:

(1) x^2+y^2 = ...

(2) xy = ...

(3) Prove that: $\frac{x^2+y^2}{xy}=38$

....

....

[9] Complete:

(1)	$(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})=$ ....
(2)	If : $x=3+\sqrt{2}$, then its conjugate is .... and the product of multiplying x by its conjugate is ....
(3)	The conjugate number of the number $\frac{1}{\sqrt{3}-\sqrt{2}}$ is ....
(4)	The conjugate number of the number $1+\frac{7}{\sqrt{7}}$ in the simplest form is ....
(5)	The multiplicative inverse for $(\sqrt{3}+\sqrt{2})$ in its simplest form is ....
(6)	If : $x=2+\sqrt{5}$ and y is the conjugate number of x , then $(x-y)^2=$ ....

(7)	If : $\frac{x}{5-\sqrt{5}} = 5+\sqrt{5}$, then the value of x in its simplest form is ....
(8)	If : $\frac{1}{x} = \sqrt{5}-2$, then the value of x in its simplest form is ....
(9)	If : $x = \sqrt{3}+2$, $y = \sqrt{3}-2$, then (x y , x + y) equals ....
(10)	$(\sqrt{2}+\sqrt{3})^{-9}(\sqrt{2}-\sqrt{3})^{-9} =$ ....

Sheet (9)

Operations on the cube roots

If a and b are two real numbers , then

1 $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{ab}$

For example:

- $\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt[3]{3 \times 9} = \sqrt[3]{27} = 3$
- $\sqrt[3]{2} \times \sqrt[3]{-4} = \sqrt[3]{2 \times -4} = \sqrt[3]{-8} = -2$

2 $\dfrac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\dfrac{a}{b}}$ (where $b \neq 0$)

For example:

- $\dfrac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\dfrac{32}{4}} = \sqrt[3]{8} = 2$
- $\dfrac{\sqrt[3]{54}}{\sqrt[3]{-2}} = \sqrt[3]{\dfrac{54}{-2}} = \sqrt[3]{-27} = -3$

Remarks

* If a and b are two real numbers , then :

1 $\sqrt[3]{a^3 + b^3} \neq a + b$, $\sqrt[3]{a^3 - b^3} \neq a - b$

2 $\sqrt[3]{-a} = -\sqrt[3]{a}$

3 $a\sqrt[3]{b} = \sqrt[3]{a^3 b}$

For example :
- $3\sqrt[3]{\dfrac{1}{9}} = \sqrt[3]{27 \times \dfrac{1}{9}} = \sqrt[3]{3}$
- $8\sqrt[3]{\dfrac{1}{4}} = 4 \times 2\sqrt[3]{\dfrac{1}{4}} = 4\sqrt[3]{8 \times \dfrac{1}{4}} = 4\sqrt[3]{2}$

4 $\sqrt[3]{\dfrac{a}{b}} = \sqrt[3]{\dfrac{a}{b} \times \dfrac{b^2}{b^2}} = \sqrt[3]{\dfrac{ab^2}{b^3}} = \dfrac{1}{b}\sqrt[3]{ab^2}$

For example :
- $\sqrt[3]{\dfrac{1}{3}} = \sqrt[3]{\dfrac{1}{3} \times \dfrac{9}{9}} = \sqrt[3]{\dfrac{9}{27}} = \dfrac{1}{3}\sqrt[3]{9}$

Important Remarks

$\sqrt[3]{16} = \sqrt[3]{8} \times \sqrt[3]{2} = 2\sqrt[3]{2}$

$\sqrt[3]{24} = \sqrt[3]{8} \times \sqrt[3]{3} = 2\sqrt[3]{3}$

$\sqrt[3]{54} = \sqrt[3]{27} \times \sqrt[3]{2} = 3\sqrt[3]{2}$

$\sqrt[3]{81} = \sqrt[3]{27} \times \sqrt[3]{3} = 3\sqrt[3]{3}$

$\sqrt[3]{128} = \sqrt[3]{64} \times \sqrt[3]{2} = 4\sqrt[3]{2}$

$\sqrt[3]{40} = \sqrt[3]{8} \times \sqrt[3]{5} = 2\sqrt[3]{5}$

$\sqrt[3]{250} = \sqrt[3]{125} \times \sqrt[3]{2} = 5\sqrt[3]{2}$

$\sqrt[3]{135} = \sqrt[3]{27} \times \sqrt[3]{5} = 3\sqrt[3]{5}$

[1] Find the result in its simplest form:

(1)	$\sqrt[3]{2} \times \sqrt[3]{32}$
(2)	$\dfrac{\sqrt[3]{72}}{\sqrt[3]{9}}$
(3)	$\dfrac{4\sqrt[3]{-54}}{2\sqrt[3]{-2}}$
(4)	$\frac{1}{2}\sqrt[3]{10} \times 6\sqrt[3]{100}$
(5)	$\sqrt[3]{\frac{2}{5}} \times \sqrt[3]{\frac{4}{25}}$
(6)	$\sqrt[3]{\frac{3}{4}} \div \sqrt[3]{\frac{2}{9}}$

[2] Find the result in its simplest form:

(1)	$\sqrt[3]{16} - \sqrt[3]{2}$
(2)	$\sqrt[3]{81} + \sqrt[3]{-24}$
(3)	$2\sqrt[3]{54} - 5\sqrt[3]{2} + \sqrt[3]{16}$
(4)	$\sqrt[3]{125} - \sqrt[3]{24}$
(5)	$\sqrt[3]{54} + \sqrt[3]{16} - \sqrt[3]{250}$
(6)	$\sqrt[3]{16} - \frac{1}{3}\sqrt[3]{54} + \sqrt[3]{-2}$
(7)	$\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54}$
(8)	$\sqrt[3]{54} \times \sqrt[3]{16} \div (\sqrt[3]{4} \times 6)$

[3] Simplify each of the following:

(1) $\frac{7}{3}\sqrt{18}+\sqrt[3]{54}-7\sqrt{2}+\sqrt[3]{16}$

....

....

....

(2) $\sqrt{27}+\frac{1}{3}\sqrt[3]{27}-9\sqrt{\frac{1}{3}}-1$

....

....

....

(3) $\sqrt[3]{-16}+\frac{14}{\sqrt{7}}-\sqrt{28}+\sqrt[3]{54}$

....

....

....

(4) $\sqrt{18}+\sqrt[3]{54}-\frac{\sqrt{216}}{\sqrt{12}}-\sqrt[3]{16}$

....

....

....

(5) $5\sqrt{2}-\frac{1}{2}\sqrt{200}+(\sqrt[3]{5}\times\sqrt[3]{25})$

....

....

....

[4]

If $a = \sqrt[3]{5} + 1$, $b = \sqrt[3]{5} - 1$ **Find the value of the following :**

1 $(a - b)^5$

2 $(a + b)^3$

....

....

....

....

....

....

[5]

If $x = 3 + \sqrt[3]{6}$, $y = 3 - \sqrt[3]{6}$ **Find the value of :** $\left(\frac{x - y}{x + y}\right)^3$

....

....

....

....

....

....

[6] Complete:

(1)	$\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{-12} =$ ....
(2)	$\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt{\ldots\ldots\ldots}$

(3)	$\sqrt[3]{54}-\sqrt[3]{-16}=\sqrt[3]{\cdots\cdots\cdots}$
(4)	The conjugate of the number $\frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}}$ is ………
(5)	$[0,5]-[0,3]=$ ………
(6)	$\sqrt[3]{54}-\sqrt[3]{2}=$ ………
(7)	If : $x=\sqrt[3]{2}+1$ and $y=\sqrt[3]{2}-1$, then $(x+y)^3=$ ………
(8)	$2\sqrt{\frac{1}{2}}-\sqrt{2}=$ ………
(9)	The number $-\sqrt{11}$ is included between the two consecutive integers ……… and ……

[7] Choose the correct answer:

(1) $\sqrt[3]{54}+\sqrt[3]{-2}=$ ………

(a) $\sqrt[3]{52}$ (b) $\sqrt[3]{2}$ (c) $2\sqrt[3]{2}$ (d) $4\sqrt[3]{2}$

(2) $\sqrt[3]{-64}+\sqrt{16}=$ ………

(a) zero (b) 8 (c) -8 (d) ± 8

(3) $\frac{\sqrt[3]{16}}{\sqrt[3]{2}}=$ ………

(a) 8 (b) -2 (c) 2 (d) $2\sqrt[3]{2}$

(4) $\sqrt[3]{2}+\sqrt[3]{2}=$ ………

(a) $\sqrt[3]{2}$ (b) $\sqrt[3]{4}$ (c) $\sqrt[3]{8}$ (d) $\sqrt[3]{16}$

(5) $\sqrt[3]{\frac{2}{9}}=$ ………

(a) $\frac{\sqrt[3]{6}}{3}$ (b) $\sqrt[3]{\frac{1}{6}}$ (c) $\sqrt[3]{6}$ (d) $\sqrt[3]{2}$

(6) If : $X=]-\infty,0[$, then $\acute{X}=$ ………

(a) $\mathbb{R}_+$ (b) $[0,\infty[$ (c) $]-\infty,0]$ (d) $\mathbb{R}_-$

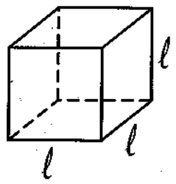
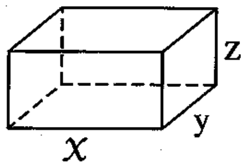
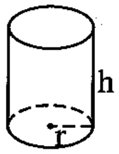
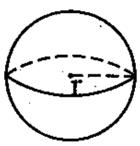
(7)	The multiplicative inverse of the number $\sqrt{\frac{3}{2}}$ is ……… (a) $\frac{2}{3}\sqrt{2}$ (b) $\frac{3\sqrt{2}}{2}$ (c) $\frac{\sqrt{6}}{3}$ (d) $-\sqrt{\frac{3}{2}}$
(8)	The irrational number in the following numbers is ……… (a) $\sqrt{\frac{1}{4}}$ (b) $\sqrt[3]{8}$ (c) $\sqrt{\frac{4}{9}}$ (d) $2\sqrt{2}$
(9)	$]-1, 3] \cap [-3, -1] =$ ……… (a) $\varnothing$ (b) $\{-3\}$ (c) $\{-1\}$ (d) $\{3\}$
(10)	If : $x = \sqrt{3} + \sqrt{2}$ and $x\,y = 1$, then y = ……… (a) $\sqrt{2} - \sqrt{3}$ (b) $\sqrt{3} + \sqrt{2}$ (c) $\sqrt{3} - \sqrt{2}$ (d) 1

Sheet (10)

Applications on the real numbers

Sheet (10) Applications on the real numbers

In the following , we will summarize the previous rules of areas and volumes of some solids :

The solid		The lateral area	The total area	The volume
The cube		$4\ell^2$	$6\ell^2$	ℓ^3
The cuboid		$2(x+y)\times z$	$2(xy+yz+zx)$	xyz
The cylinder		$2\pi rh$	$2\pi rh+2\pi r^2$ $=2\pi r(h+r)$	$\pi r^2 h$
The sphere		–	$4\pi r^2$	$\frac{4}{3}\pi r^3$

THE CUBE

[1] A cube with volume 125 cm^3. Find its total area and its lateral area.

....

....

....

[2] A cube whose lateral area is 36 cm^2. Find its total area, and its volume.

....

....

....

[3] The perimeter of one face of a cube is 12 cm. Find its volume, and its lateral area.

....

....

....

[4] The sum of lengths of all edges of a cube is 60 cm. Find its volume and its total area.

....

....

....

[5] Complete:

(1)	If the edge length of a cube is 5 cm. , then its volume = ·········· cm^3.
(2)	The edge length of a cube is 4 cm. , then its total area = ·········· cm^2.
(3)	The lateral area of a cube whose edge length is ℓ cm. = ·········· cm^2.
(4)	The cube whose volume ℓ^3 cm^3. , its total area = ·········· cm^2.
(5)	The cube whose edge length is 2ℓ , then its volume = ·········· cm^3.

[6] Choose the correct answer:

(1) The volume of a cube is 1 cm^3 , then the sum of its edge lengths = ... cm.

(a) 1 (b) 6 (c) 8 (d) 12

(2) 📖 The volume of a cube is 64 cm^3 , then its lateral area = ... cm^2

(a) 4 (b) 8 (c) 64 (d) 96

(3) If the total area of a cube is 96 cm^2 , then the area of one face = ... cm^2

(a) 16 (b) 64 (c) 24 (d) 48

(4) If the area of the six faces of a cube = 54 cm^2 , then its volume = ... cm^3

(a) 54 (b) 44 (c) 72 (d) 27

(5) If the volume of a cube = 64 cm^3 , then the length of a diagonal of one face = ... cm.

(a) 16 (b) $4\sqrt{2}$ (c) 32 (d) 64

(6) The volume of a cube is 5 cm^3 If the edge length became twice the first , then its volume = ... cm^3

(a) 10 (b) 20 (c) 30 (d) 40

(7) 📖 The edge length of a cube whose volume is $2\sqrt{2}$ cm^3 = ... cm.

(a) $\sqrt{2}$ (b) 2 (c) 8 (d) 1.5

THE CUBOID

[1] The dimensions of the base of a cuboid are 9 cm and 10 cm and its height is 5 cm. Find its volume, its lateral area and its total area.

....

....

....

[2] The height of a cuboid is 4 cm and its base is a square of side length 5 cm. Find its volume, its lateral area and its total area.

....

....

....

[3] The lateral area of a cuboid is 480 cm^2 and its base is in the shape of a square whose side length is 10 cm. Calculate its height.

....

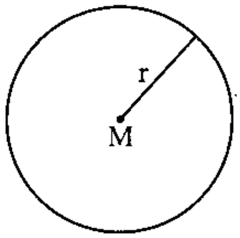
....

....

THE CIRCLE

If M is a circle with radius length r , then :

1. The circumference of the circle = $2\pi r$ length unit.
2. The area of the circle = πr^2 square unit.



[1] A circle is of radius length 10.5 cm. Find each of its circumference and its area. $\left(\pi = \frac{22}{7}\right)$

....

....

[2] The area of a circle is 25π cm^2. Calculate its circumference in terms of π. $\left(\pi = \frac{22}{7}\right)$

....

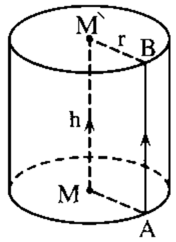
....

[3] The area of a circle is 154 cm^2. Find its circumference and its diameter length. $\left(\pi = \frac{22}{7}\right)$

....

....

THE RIGHT CIRCULAR CYLINDER



1 The lateral area of the cylinder = $2 \pi r h$ square unit.

2 The total area of the cylinder = the lateral area of the cylinder + twice the area of the base
$= 2 \pi r h + 2 \pi r^2$ square unit.

3 The volume of the cylinder = the area of the base × height = $\pi r^2 h$ cube unit.

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

(1) A right circular cylinder , the radius length of its base is 14 cm. and its height is 20 cm. Find the volume and the total area of the cylinder. « 12320 cm^3 , 2992 cm^2 »

....

....

(2) Find the lateral area for a right circular cylinder of volume 924 cm^3 , and of a height 6 cm. « 264 cm^2 »

....

....

(3) Find the total area of a right circular cylinder of volume 7536 cm^3 and its height is 24 cm. ($\pi = 3.14$) « 2135.2 cm^2 »

....

....

(4) Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is 72π cm^3 « $2\sqrt[3]{9}$ cm. »

....

....

THE SPHERE

1 The area of the sphere = $4 \pi r^2$ square unit.

2 The volume of the sphere = $\frac{4}{3} \pi r^3$ cube unit.

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

(1) Find the volume and the surface area of a sphere if the length of its diameter is 4.2 cm.

« 38.808 cm.^{3} , 55.44 cm.^{2} »

....

....

(2) The volume of a sphere is 4188 cm.^{3} Find its radius length. ($\pi = 3.141$) « 10 cm. »

....

....

(3) **The volume of a sphere = $\frac{500}{3} \pi$ cm.^{3} Find the length of its diameter.**

....

....

(4) 🕮 The volume of a sphere is 562.5π cm.^{3}

Find its surface area in terms of π « 225π »

....

....

General Revision on Applications on the Real Numbers

[1] Complete:

(1)	The sphere whose volume = 36π cm^3 has a radius length = ... cm.
(2)	A right circular cylinder, its volume is 343π cm^3. If its height equals its base radius length, then its height = ... cm.
(3)	The volume of a cube is 64 cm^3, then its total area is .... cm^3.

[2] Choose the correct answer:

(1)	The circle whose radius length = $\sqrt{14}$ cm. has an area = ... cm^2. (a) 14π (b) $2\sqrt{14}\pi$ (c) 14 (d) $2\sqrt{14}$
(2)	$]2, 4[\cup \{2, 4\}$ = ... (a) $\varnothing$ (b) $\{2, 4\}$ (c) $[2, 4]$ (d) $]2, 4[$
(3)	The right circular cylinder whose base radius length = 3 cm. and its height = 5 cm. its volume = ... cm^3. (a) 15π (b) 75π (c) 45π (d) $\frac{3}{5}\pi$
(4)	If : $-\sqrt{25} = \sqrt[3]{x}$, then x = ... (a) -5 (b) -25 (c) -125 (d) 125
(5)	🕮 The volume of the sphere whose diameter length is 6 cm. = ... cm^3. (a) 288 (b) 12π (c) 36π (d) 288π
(6)	If the volume of a sphere = $\frac{9}{16}\pi$ cm^3, then its radius length = ... cm. (a) 3 (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$
(7)	If the surface area of a sphere is 9π cm^2, then its diameter length = ... cm. (a) 9 (b) 3 (c) 1.5 (d) 6
(8)	If three quarters of volume of a sphere equals 8π cm^3, then the length of its radius equals .... cm. (a) 64 (b) 8 (c) 4 (d) 2

Sheet (11)

Solving equations and and inequalities of the first degree in one variable in R

Sheet (11)
Solving equations and and inequalities of the first degree in one variable in R

[1] Find the solution set for each of the following equations in R, then graph the solution on the number line:

(1) $x+5=0$

....

....

....

(2) $5x+6=1$

....

....

....

(3) $2x+4=3$

....

....

....

....

(4) $2x-3=4$

....

....

....

....

(5) $\sqrt{5}x-1=4$

....

....

....

(6) $x-1=\sqrt{3}$

....

....

....

(7) $\sqrt{3}x-1=2$

....

....

....

....

(8) $7x-\sqrt{7}=6\sqrt{7}$

....

....

....

....

(9) $x-\sqrt{5}=1$

……………………………………

……………………………………

……………………………………

……………………………………

[2] Find the solution set for each of the following inequalities in R in the form of interval, then graph the solution on the number line:

(1) $2x>6$

……………………………………

……………………………………

……………………………………

……………………………………

(2) $-7x\geq-14$

……………………………………

……………………………………

……………………………………

……………………………………

(3) $x+3\leq 5$

……………………………………

……………………………………

……………………………………

……………………………………

(4) $5-x>3$

……………………………………

……………………………………

……………………………………

……………………………………

(5) $2x+5\geq 3$

……………………………………

……………………………………

……………………………………

……………………………………

(6) $1-5x<6$

……………………………………

……………………………………

……………………………………

……………………………………

(7) $\frac{1}{2}x+1\le 2$

…………………………………………

…………………………………………

…………………………………………

…………………………………………

(8) $3-2x\le 7$

…………………………………………

…………………………………………

…………………………………………

…………………………………………

(9) $3<x+2\le 6$

…………………………………………

…………………………………………

…………………………………………

…………………………………………

(10) $-5<x+3<9$

…………………………………………

…………………………………………

…………………………………………

…………………………………………

(11) $-3\le -x\le 3$

…………………………………………

…………………………………………

…………………………………………

…………………………………………

(12) $1<5-x\le 3$

…………………………………………

…………………………………………

…………………………………………

…………………………………………

(13) $\sqrt[3]{-8}\le x+1\le \sqrt{9}$

…………………………………………

…………………………………………

…………………………………………

…………………………………………

(14) $5<3-x\le 3^2$

…………………………………………

…………………………………………

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…………………………………………

(15) $|-3|<2x-1<5$

(16) $3x<2x+4$

………………………………………… …………………………………………
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………………………………………… …………………………………………
………………………………………… …………………………………………

(17) $7x - 9 \geq 4x$

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…………………………………………
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…………………………………………

(18) $5x - 3 < 2x + 9$

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…………………………………………
…………………………………………

(19) $x + 3 \geq 2x \geq x - 2$

…………………………………………
…………………………………………
…………………………………………
…………………………………………

(20) $4x \leq 5x + 2 \leq 4x + 3$

…………………………………………
…………………………………………
…………………………………………
…………………………………………

(21) $x - 1 < 3x - 1 \leq x + 1$

…………………………………………
…………………………………………
…………………………………………
…………………………………………

Homework

(1) $1 \le 3 - 2x < 5$

....

....

....

....

(2) $2x - 1 \ge 7$

....

....

....

....

(3) $-7 \le 3x + 2 < 11$

....

....

....

....

(4) $x + 1 \le 2x - 3 < x + 4$

....

....

....

....

(5) $-2 < 3x + 7 \le 10$

....

....

....

....

(6) $-3 < 2x - 3 \le \sqrt[3]{125}$

....

....

....

....

(7) $3 \le 2x - 1 < 11$

....

....

....

....

(8) $6 < 2x + 4 \le 10$

....

....

....

....

(9) $5 < 2x - 3 \le 11$

(10) $x + 4 \ge 2x - 3 > x + 1$

……………………………………… | ………………………………………

……………………………………… | ………………………………………

……………………………………… | ………………………………………

……………………………………… | ………………………………………

(11) $-2 < 3x + 1 \le 10$

………………………………………

………………………………………

………………………………………

………………………………………

(12) $-2 < 3x + 7 \le 10$

………………………………………

………………………………………

………………………………………

………………………………………

[1] Complete:

(1)	If $x - 3 \ge 0$, then x ……….
(2)	If $5x < 15$, then x ……….
(3)	If $1 - x > 4$, then x ……….
(4)	If $-2x \le 3$, then x ……….
(5)	If $\sqrt{2}\, x \le 4$, then x ……….
(6)	The S.S. of the inequality $-5 \le -x < 2$ in $\mathbb{R}$ is ……….

[2] Choose the correct answer:

(1) The S.S. of the inequality : $x + 3 < 3$ in $\mathbb{R}$ is ……….

(a) $]-\infty , 0[$ (b) $]-\infty , 0]$ (c) $[0 , \infty[$ (d) $]0 , \infty[$

(2)	The S.S. of the inequality : $1 > x - 5 > -1$ in $\mathbb{R}$ is ………. (a) $[4 , 6]$ (b) $]4 , 6[$ (c) $]4 , 6]$ (d) $[4 , 6[$
(3)	If $x > 5$, then $-x$ ………. (a) < -9 (b) ≥ -5 (c) < -5 (d) > -5
(4)	If $-2 < x < 2$, then $2x + 3$ belongs to ………. (a) $[-1 , 7]$ (b) $]-1 , 5[$ (c) $]-1 , 7[$ (d) $]-4 , 6[$
(5)	The S.S. of the inequality : $-1 < x + 3 < 3$ in $\mathbb{R}$ is ………. (a) $[-4 , 0]$ (b) $[2 , 6]$ (c) $]-4 , 0[$ (d) $]2 , 6[$
(6)	The S.S. of the inequality : $-2x \geq 6$ in $\mathbb{R}$ is ………. (a) $]-8 , -3[$ (b) $]-\infty , -3]$ (c) $[-3 , \infty[$ (d) $]-3 , \infty[$
(7)	The S.S. of the inequality : $3 \leq x + 2 < 5$ in $\mathbb{R}$ equals ………. (a) $[1 , 3[$ (b) $]1 , 3]$ (c) $[1 , 3]$ (d) $]1 , 3[$
(8)	The S.S. in $\mathbb{R}$ of the inequality $-1 < -x \leq 1$ is ………. (a) $]-1 , 1]$ (b) $[-1 , 1]$ (c) $[-1 , 1[$ (d) $]-1 , 1[$

Sheet (12)

Relation between two variables

Sheet (12) Relation between two variables

Islam has 50 pounds. If Islam went to the playhouse , he would find two kinds of favourite games :

- The first kind costs 5 pounds for the game each time.
- The second kind costs 10 pounds for the game each time.
- Assume that the number of times that he will play the first kind of games is x and of the second kind is y
- Then , the cost of playing the first game is $5x$ pounds and the cost of playing the second game is $10y$ pounds.
- In order to spend all his money , it should be : $5x + 10y = 50$

We can simplify the previous relation by dividing all terms by 5 to get an equivalent equation which is $x + 2y = 10$ it can be written also in the form : $2y = 10 - x$

$$y = \frac{10 - x}{2}$$

If Islam decided that he will not play the first kind.

$x = 0$, then $y = \frac{10-0}{2} = 5$

If he decided to play two times of the first kind

$x = 2$, then $y = \frac{10-2}{2} = 4$

(1) **Complete the following ordered pairs which satisfy the relation : $y = 3x - 1$**

(5 , ……….) , (2 , ……….) , (0 , ……….) , (– 3 , ……….)

(2) **Show which of the following ordered pairs satisfy the relation : $y - 4x = 7$**

1 (1 , 2) 2 (3 , – 5) 3 (– 1 , 3)

(3) Find four ordered pairs satisfying the relation $y = 2x - 1$ and represent it.

(4) Graph the relation $y = 2x - 3$

(5)	Represent graphically the relation $x+y=2$
(6)	Represent the relation $2y-x=2$ graphically
(7)	Find three ordered pairs that satisfy the relation $x+2y=6$, then represent it graphically.

[8] Complete:

(1)	If (2,3) satisfies the relation $x+y=k$, then k = ………
(2)	If (k,2k) satisfies the relation $x+y-15=0$, then k = ………
(3)	If (-2,k) satisfies the relation $2x+3y=35$, then k = ………

[9] Choose the correct answer:

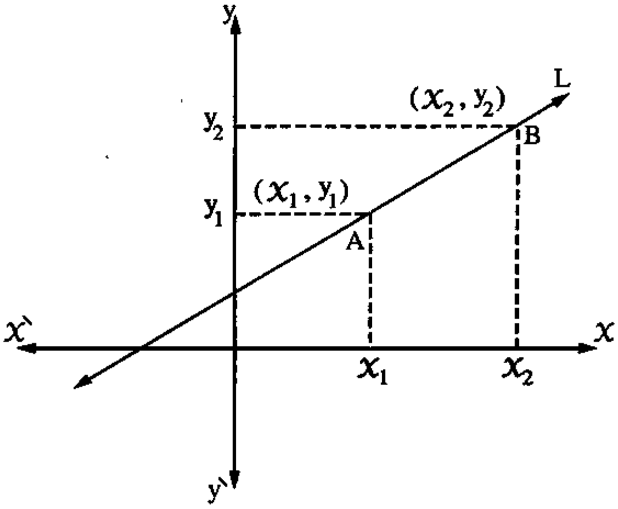
(1)	If $(2, -5)$ satisfies the relation : $3x - y + c = 0$, then c = ………. (a) 1 (b) – 1 (c) 11 (d) – 11
(2)	Which of the following ordered pairs satisfies the relation : $2x + y = 5$? (a) (– 1 , 3) (b) (1 , 3) (c) (3 , 1) (d) (2 , 2)
(3)	(3 , 2) does not satisfy the relation ………. (a) $y + x = 5$ (b) $3y - x = 3$ (c) $y + x = 7$ (d) $x - y = 1$
(4)	The relation : $3x + 8y = 24$ is represented by a straight line intersecting y-axis at the point ………. (a) (0 , 8) (b) (8 , 0) (c) (0 , 3) (d) (3 , 0)

(5)	The relation $2x + 7y = 14$ is represented by a straight line intersecting x-axis at the point ……… (a) (2 , 0) (b) (0 , 2) (c) (7 , 0) (d) (0 , 7)
(6)	The image of the point (5 , 3) by reflection in the straight line $x = 1$ is ……… (a) (5 , – 3) (b) (– 5 , 3) (c) (– 3 , 3) (d) (6 , 3)
(7)	The image of the point (4 , – 2) by reflection in the straight line y = 1 is ……… (a) (4 , 2) (b) (– 4 , – 2) (c) (4 , – 1) (d) (4 , 4)
(8)	If : (2 k , 3 k) satisfies the relation $x + y = 15$, then k = ……… (a) 5 (b) 3 (c) – 5 (d) – 3

Homework

(1)	Write two ordered pairs satisfying the relation $y = x + 1$
(2)	Represent graphically the relation $y = 2 - x$
(3)	If (-1,5) satisfies the relation $3x + ky = 7$, then k = ………

Sheet (13)
Slope of straight line



If a point moves on a straight line L from the location A (x_1, y_1) to the location B (x_2, y_2), then :

The change in the x-coordinates = $x_2 - x_1$

It is called (the horizontal change).

The change in the y-coordinates = $y_2 - y_1$

It is called (the vertical change).

The ratio of the change in the y-coordinates to the change in the x-coordinates is called the slope of the straight line (m).

$$\text{The slope of the straight line} = \frac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$$

i.e. $S = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$, S is undefined if $x_1 = x_2$

1 **Classify the slope of the straight line in each of the following figures showing whether it is (positive – negative – zero – undefined)**

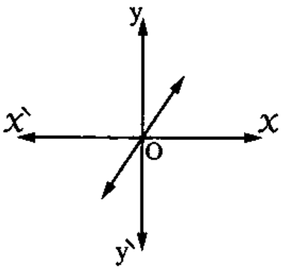


Fig. (1)

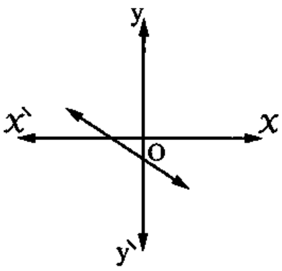


Fig. (2)

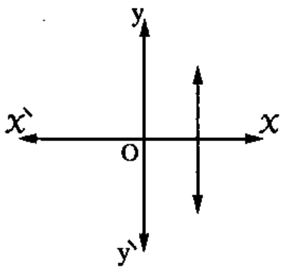


Fig. (3)

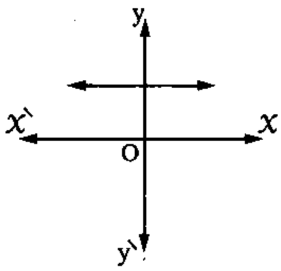


Fig. (4)

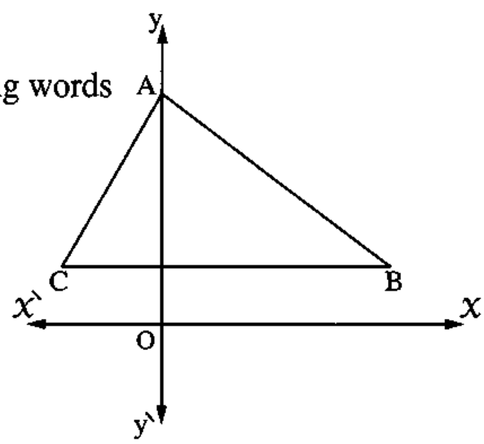
[2] Find the slope of the straight line passing through the two points in each of the following:

(1)	A (1 , 3) , B (3 , 4)
(2)	A (1 , 2) , B (5 , 0)
(3)	A (2 , – 1) , B (4 , – 1)
(4)	A (5 , 2) , B (5 , 4)
(5)	A (3,5) , B (5,-1)
(6)	A (-1,3) , B (2,4)
(7)	A (1,3) , B (2,1)
(8)	A (1,3) , B (2,3)

[3]

In the opposite figure :

ABC is a triangle. Complete by using one of the following words (positive , negative , zero , undefined)



1 The slope of $\overleftrightarrow{AB}$ is ....

2 The slope of $\overleftrightarrow{BC}$ is ....

3 The slope of $\overleftrightarrow{AO}$ is ....

4 The slope of $\overleftrightarrow{AC}$ is ....

[4] Complete:

(1)	The slope of any horizontal straight line equals ....
(2)	The slope of any straight line parallel to y-axis is ....
(3)	The straight line whose slope = zero is parallel to ....
(4)	If A , B and C are collinear then the slope of $\overleftrightarrow{AB}$ = the slope of ....
(5)	The slope of the straight line $\overleftrightarrow{AB}$ where A (2 , 3) and B (0 , 4) is ....
(6)	If the slope of the straight line which passes through the two points (1 , 3) , (3 , k) equals 3 , find the value of k « 9 »
(7)	If the slope of the straight line which passes through the two points (3 , c) and (5 , – 2) equals – 3 , find the value of c « 4 »

Sheet (14)

The ascending and descending cumulative frequency table and their graphical representation

Example 1

The following frequency table shows the weekly wages in pounds of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the ascending cumulative frequency table and represent it graphically , then find :

1. The number of workers whose weekly wages are less than 60 pounds.
2. The percentage of the number of workers whose weekly wages are less than 60 pounds.

Solution

- Form the ascending cumulative frequency table as follows :

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (Frequency)	5	12	22	7	4

Less than 54 = 0
Less than 58 = 5 + 0 = 5
Less than 62 = 5 + 12 = 17
Less than 66 = 5 + 12 + 22 = 39
Less than 70 = 5 + 12 + 22 + 7 = 46
Less than 74 = 5 + 12 + 22 + 7 + 4 = 50

The upper boundaries of sets	Frequency
Less than 54	zero
Less than 58	5
Less than 62	17
Less than 66	39
Less than 70	46
Less than 74	50

The ascending cumulative frequency table.

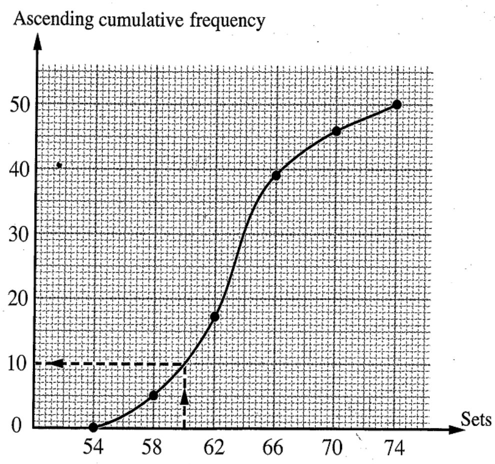
Notice that : The ascending cumulative frequency begins with zero and ends at the total frequency.

To represent the ascending cumulative frequency table graphically , do as follows :

1. Specialize the horizontal axis for sets and the vertical axis for the ascending cumulative frequency.
2. Choose a suitable scale to represent data on the vertical axis so that it contains the ascending cumulative frequency easily.
3. Represent the ascending cumulative frequency of each set , then draw the graph (the curve) such that it passes through the points which we located as shown in the opposite figure.

- From the graph , we find that :

1. The number of workers whose weekly wages are less than 60 pounds = 10 workers.
2. The percentage of the number of workers whose weekly wages are less than 60 pounds $= \frac{10}{50} \times 100\% = 20\%$



The assending cumulative frequency curve

Example 2

The following frequency table shows the weekly wages of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the descending cumulative frequency table and represent it graphically , then find :

1 The number of workers whose weekly wages are 60 pounds or more.

2 The percentage of the number of workers whose weekly wages are 60 pounds or more.

Solution

- Form the descending cumulative frequency table as follows :

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (Frequency)	5	12	22	7	4

54 and more = 5 + 12 + 22 + 7 + 4 = 50

58 and more = 12 + 22 + 7 + 4 = 45

62 and more = 22 + 7 + 4 = 33

66 and more = 7 + 4 = 11

70 and more = 4

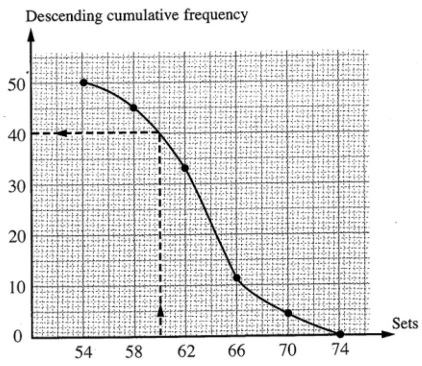
74 and more = 0

The lower boundaries of sets	Frequency
54 and more	50
58 and more	45
62 and more	33
66 and more	11
70 and more	4
74 and more	zero

The descending cumulative frequency table

Notice that : The descending cumulative frequency begins with the total frequency and ends with zero.

- To represent this table graphically , follow the same previous steps in the ascending cumulative frequency table to get the opposite graph.



The descending cumulative frequency curve

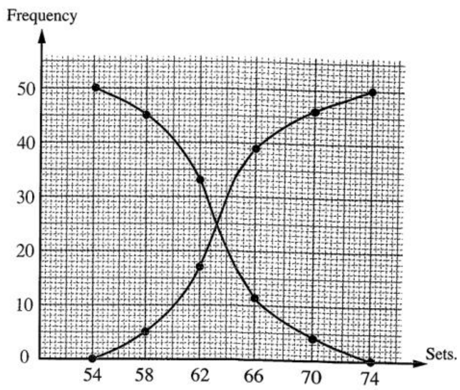
- From the graph , we find that :

1 The number of workers whose weekly wages are 60 pounds or more = 40 workers.

2 The percentage of those workers $= \frac{40}{50} \times 100\% = 80\%$

Remark

we can graph the two curves of the ascending and descending cumulative frequency of a frequency distribution in one sketch as shown in the opposite graph.



Sheet (15)
The Mean

Remember that :

To calculate the mean of a set of values, do as follows :

1 Find the sum of these values. **2** Divide this sum by the number of these values

i.e. The mean of a set of values $= \dfrac{\text{The sum of values}}{\text{Number of values}}$

For example:

If the marks of 5 students are 25 , 23 , 21 , 22 , 24

, then the mean of marks $= \dfrac{25 + 23 + 21 + 22 + 24}{5} = 23$ marks.

Notice that : $23 \times 5 = 115$

, the sum of marks of the 5 students $= 25 + 23 + 21 + 22 + 24 = 115$

i.e. The mean is the value which is given to each item of a set , then the sum of these new values is the same sum of the original values.

Finding the mean of data from the frequency table with sets

Example **The following table shows the distribution of the marks of 50 students in mathematics :**

Sets	10 –	20 –	30 –	40 –	50 –	**Total**
Frequency	8	12	14	9	7	**50**

Find the mean of these marks.

1 Determine the centres of sets according to the rule :

$$\text{The centre of a set} = \frac{\text{the lower limit} + \text{the upper limit}}{2}$$

, then the centre of the first set $= \frac{10 + 20}{2} = 15$

, the centre of the second set $= \frac{20 + 30}{2} = 25$ ... and so on.

Since the lengths of the subsets are equal and each of them = 10 therefore we consider the upper limit of the last set = 60

, then its centre $= \frac{50 + 60}{2} = 55$

2 **Form the vertical table :**

Set	Centre of the set « X »	Frequency « f »	$X \times f$
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

3 The mean $= \frac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \frac{1700}{50} = 34$ marks.

Sheet (16)
The Median

Remember that

The median is the middle value in a set of values after arranging it ascendingly or descendingly. such that the number of values which are less than it is equal to the number of values which are greater than it.

- To find the median of a set of values , we do as follows :

We arrange the values ascendingly or descendingly

If the values number is odd , then

The median is the value lying in the middle exactly.

For example :
If the values are
42 , 23 , 17 , 30 and 20
We arrange them ascendingly as follows
17 , 20 , 23 , 30 , 42

The median = 23

If the values number is even , then

The median $= \frac{\text{The sum of the two values lying in the middle}}{2}$

For example :
If the values are
27 , 13 , 23 , 24 , 13 , 21
We arrange them ascendingly as follows
13 , 13 , $\boxed{21 , 23}$, 24 , 27

The median $= \frac{21 + 23}{2} = 22$

Finding the median of a frequency distribution with sets graphically

Example **The following table shows the frequency distribution of marks of 50 students in math exam :**

Sets of marks	0 –	10 –	20 –	30 –	40 –	50 –	Total
Number of students	2	5	8	19	14	2	50

Find the median mark of the student.

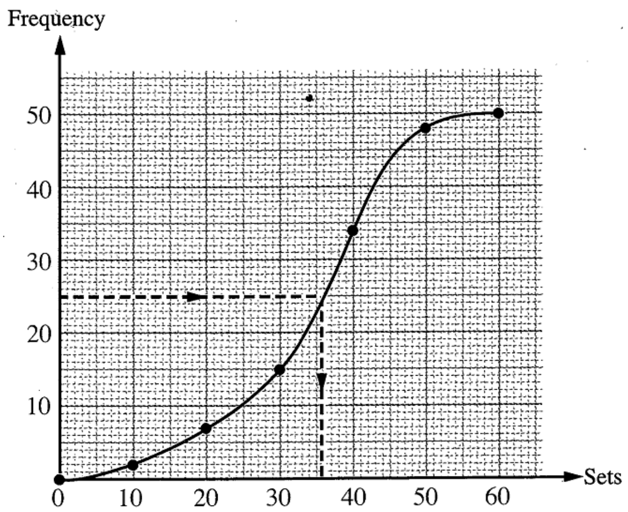
Solution

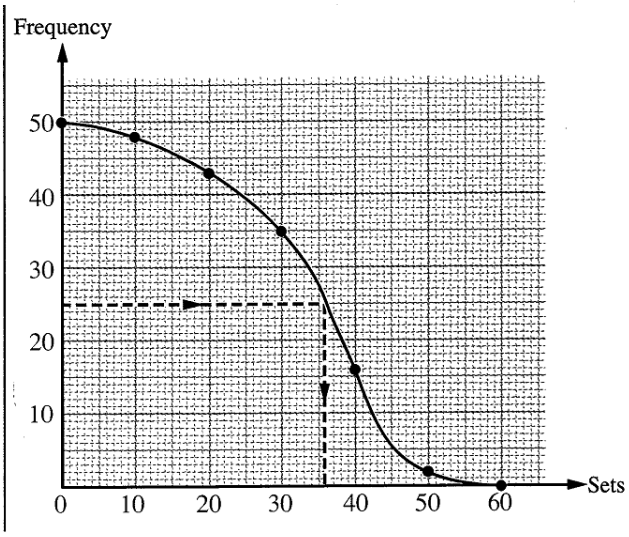
Using the ascending cumulative frequency curve :

The upper boundaries of sets	Frequency
Less than 0	0
Less than 10	2
Less than 20	7
Less than 30	15
Less than 40	34
Less than 50	48
Less than 60	50

Using the descending cumulative frequency curve :

The lower boundaries of sets	Frequency
0 and more	50
10 and more	48
20 and more	43
30 and more	35
40 and more	16
50 and more	2
60 and more	0





$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ From the two previous graphs , the median = 36 approximately

Sheet (17)
The Mode

Remember that

The mode of a set of values is the most common value in the set, or in other words, it is the value which is repeated more than any other values.

For example :

The mode of the set of the values : 7 , 3 , 4 , 1 , 7 , 9 , 7 , 4 is 7

Finding the mode for a frequency distribution with equal sets in range.

The following is an example which shows how to find the mode of a frequency distribution with sets.

Example

The following is the frequency distribution of marks of 100 pupils in one of the exams :

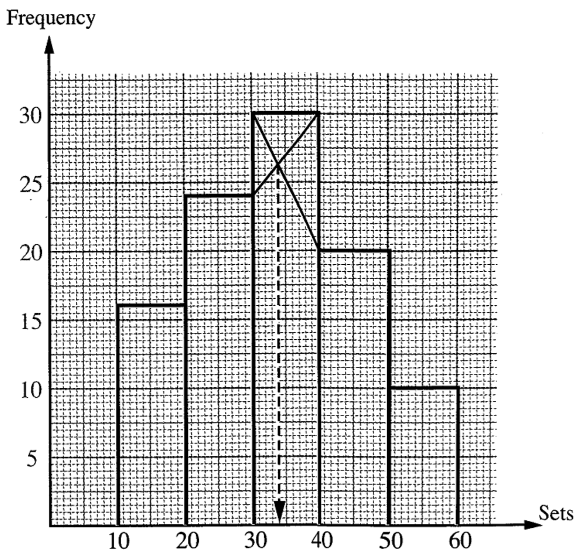
Set of marks	10 –	20 –	30 –	40 –	50 –	**Total**
Number of pupils	16	24	30	20	10	**100**

Find the mode mark for these pupils.

Solution

We can find the mode of that distribution graphically using the histogram as follows :

1. Draw two orthogonal axes : one of them is horizontal and the other is vertical to represent the frequency of each set.
2. Divide the horizontal axis into a number of equal parts with a suitable drawing scale to represent the sets.
3. Divide the vertical axis into a number of equal parts with a suitable drawing scale to represent the greatest frequency in the sets.
4. Draw a rectangle whose base is set (10 –) and its height equals the frequency (16)
5. Draw a second rectangle adjacent to the first one whose base is set (20 –) and its height equals the frequency (24)



[1] Choose the correct answer:

(1)	The arithmetic mean of : 3 , 10 , 2 is …………… (a) 10 (b) 5 (c) 3 (d) 6
(2)	The mean of the values : 2 , 5 , 4 , 5 is …………… (a) 4 (b) 5 (c) 16 (d) 8
(3)	The mean of the values : 2 , 8 , 6 , 4 is …………… (a) 2 (b) 5 (c) 4 (d) 6
(4)	The arithmetic mean of : 3 , 7 , 28 , 52 , 10 = …………… (a) 17 (b) 19 (c) 20 (d) 27
(5)	The arithmetic mean of the values : 19 , 32 , 21 , 6 , 12 is …………… (a) 90 (b) 32 (c) 18 (d) 6
(6)	The mean of the values : 7 , 15 , 19 , 14 and 15 is ……… (a) 14 (b) 15 (c) 16 (d) 17
(7)	The arithmetic mean of the values : 30 , 23 , 25 , 30 , 22 is ……… (a) 22 (b) 23 (c) 24 (d) 26
(8)	If the arithmetic mean of the values : 27 , 8 , 16 , 24 , 6 and k is 14 , then k = ……… (a) 3 (b) 6 (c) 27 (d) 84
(9)	If the mean of marks of 5 pupils is 20 , then the total of their marks = ………… m (a) 4 (b) 15 (c) 25 (d) 100
(10)	If the sum of 5 numbers equals 30 , then the arithmetic mean of these numbers is …… (a) 150 (b) 6 (c) 18 (d) 72
(11)	The set which its lower boundary is 2 and its upper boundary is 6 , then its centre is ……… (a) 2 (b) 6 (c) 4 (d) 8
(12)	The lowest limit of a set is 4 and the other limit is 8 , then its centre is …………… (a) 2 (b) 4 (c) 6 (d) 8
(13)	If the lower limit of a set is 6 and the upper limit is 10 , then its centre is ……… (a) 4 (b) 6 (c) 10 (d) 8
(14)	If the upper limit of a set is 19 and the lower limit of the same set is 11 , then centre is …………… (a) 10 (b) 15 (c) 20 (d) 30

(15) If the lowest boundary of a set is 10 and the upper boundary is X and its centre then X = ……….

(a) 10 (b) 15 (c) 20 (d) 30

(16) If the lower limit of a set is 18 and its centre is 20 , then its length is ……………

(a) 2 (b) 19 (c) 22 (d) 4

(17) The arithmetic mean of the values : 3 – a , 5 , 1 , 4 , 2 + a equals ……….

(a) 1 (b) 2 (c) 3 (d) 15

(18) If the arithmetic mean of the values : 9 , 6 , 5 , 14 , k is 7, then k = ……….

(a) 1 (b) 5 (c) 34 (d) 35

(19) The mean of the values : 2 – a , 4 , 1 , 5 , 3 + a is ………..

(a) 1 (b) 2 (c) 3 (d) 15

(20) The order of the median of the set of values : 8 , 4 , 7 , 6 , 5 is ……….

(a) 7 (b) 6 (c) 3 (d) 5

(21) The order of the median of the set of values : 4 , 5 , 6 , 7 and 8 is ……….

(a) third. (b) fourth. (c) fifth. (d) sixth.

(22) If the order of the median of a set of values is the fourth , then the number of these values is ……….

(a) 3 (b) 5 (c) 7 (d) 9

(23) If the median of the set of the values : 27 , 45 , 19 , 24 and 28 is X , then X = ……….

(a) 24 (b) 27 (c) 28 (d) 45

(24) The median of the values : 1 , 2 , 5 , 3 and 4 is ……………

(a) 3 (b) 4 (c) 5 (d) 2

(25) The median of the values : 2 , 9 , 3 , 7 , 5 is ……………

(a) 5 (b) 6 (c) 7 (d) 8

(26) The median of the values : 3 , 7 , 5 , 8 , 2 is ……………

(a) 3 (b) 5 (c) 8 (d) 7

(27) The median of the values : 7 , 2 , 3 , 5 , 4 is ……………

(a) 3 (b) 4 (c) 5 (d) 7

(28) The median for the values 3 , 9 , 7 , 4 and 5 is ……………

(a) 5 (b) 4 (c) 7 (d) 9

(29) The median of the set of the values : 3 , 6 , 6 , 7 , 9 , 11 , 13 , 14 , 15 and 20 is ……….

(a) 9 (b) 10 (c) 11 (d) 20

(30)	The median of values : 4 , 8 , 3 , 5 , 7 , 9 is .... (a) 5 (b) 6 (c) 7 (d) 8
(31)	The median of the set of the values : 15 , 22 , 9 , 11 and 33 is .... (a) 9 (b) 15 (c) 18 (d) 90
(32)	The median of the values : 10 , 9 , 11 , 19 , 12 is .... (a) 9 (b) 10 (c) 11 (d) 19
(33)	The median of the set of the values : 15 , 22 , 9 , 11 and 33 is .... (a) 9 (b) 15 (c) 18 (d) 90
(34)	The median of the values : 34 , 23 , 25 , 40 , 22 , 14 is .... (a) 22 (b) 33 (c) 24 (d) 25
(35)	The median of the values : 41 , 23 , 15 , 30 , 20 is .... (a) 23 (b) 15 (c) 30 (d) 20
(36)	The mode of the values : 3 , 5 , 3 , 6 , 3 and 8 is .... (a) 3 (b) 5 (c) 6 (d) 8
(37)	The mode of the sets of values : 14 , 11 , 10 , 11 , 14 , 15 , 11 is .... (a) 14 (b) 11 (c) 15 (d) 10
(38)	If the mode of the set of the values : 4 , 11 , 8 , $2X$ is 4 , then X = ... (a) 2 (b) 4 (c) 6 (d) 8
(39)	The mode of the values : 15 , 9 , $X+1$, 9 , 15 is 9 , then X = ... (a) 9 (b) 14 (c) 10 (d) 8
(40)	The mode of 7 , 8 , 9 , $X+2$ and 6 is 9 then X = ... (a) 4 (b) 5 (c) 6 (d) 7
(41)	The mode of : 5 , 6 , 7 , $X+2$ and 8 is 7 , then X = ... (a) 7 (b) 6 (c) 4 (d) 5
(42)	If the mode of the set of values : 4 , 11 , $X+3$, 6 is 6 , then X = ... (a) 2 (b) 3 (c) 4 (d) 6
(43)	The mode of the set of values : 5 , 9 , 5 , $X-2$, 9 is 9 , then X = ... (a) 5 (b) 57 (c) 9 (d) 11

[2] Essay problems:

1 Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	10	12	10	5	40

2 Find the mean of the following data :

Sets	8 –	12 –	16 –	20 –	24 –	Total
Frequency	4	10	16	12	8	50

3 The following table shows frequency distribution of marks of 32 students in an exam :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	3	6	10	8	5	32

Find the mean of this distribution.

4 The following table shows the frequency distribution , find the arithmetic mean :

Sets	10 –	20 –	30 –	40 –	50 –
Frequency	10	20	25	30	15

Homework

[1] Essay problems:

1 Find the arithmetic mean of the following frequency distribution :

Sets	1 –	3 –	5 –	7 –	9 –	Total
Frequency	4	6	8	7	5	30

2 Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	10	12	10	5	40

3 Using the following frequency distribution to find the mean :

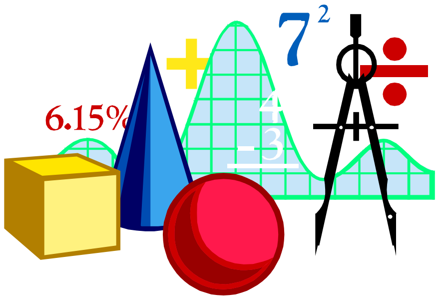
Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

[2] Complete:

1	The most common value in a set is called ....
2	The value which is the most common of a set of values is called ....
3	The mode of a set of values is ....
4	The mode of the values : 2 , 5 , 1 , 4 , 2 is ....
5	The mode of the values : 4 , 7 , 5 , 7 , 6 , 8 , 7 , 5 is ....
6	The mode of the values : 8 , 7 , 8 , 7 , 6 , 5 , 8 is ....
7	The mode of the set of values : 13 , 12 , 4 , 13 is ....
8	The mode of the set of the values : 14 , 11 , 10 , 11 , 14 , 15 , 11 is ....
9	The mode of the values : 11 , 13 , 11 , 14 , 11 , 12 is ....
10	The mode of the set of the values : 14 , 11 , 15 , 11 , 14 , 15 , 11 is ....
11	The mode value of : 13 , 23 , 46 , 33 , 46 , 43 , 33 , 46 , 32 is .... cm.
12	If the mode of the set of the values : 4 , 5 , a , 3 is 4 , then a = ...
13	If the mode of the values : 3 , 6 , a , 2 , 5 is 6 , than a = ...
14	If the mode of the set of the values : 4 , 5 , a and 3 is 3 , then a = ...
15	If the mode of the values : 5 , 7 and $x + 1$ is 7 , then x = ...
16	The mode of the values : 14 , 8 , $x + 1$, 8 , 14 is 8 , then x = ...
17	If the mode of the values : 12 , 7 , $x + 1$, 7 , 12 is 7 , then x = ...
18	If the mode of the set of the values : 15 , 9 , $x + 1$, 9 and 15 is 9 , then x = ...
19	If the mode of the set of the values : 15 , 9 , $x + 6$, 9 and 15 is 9 , then x = ...
20	If the mode of the values : 4 , 11 , 8 , and 2^x is 4 , then x = ...

GEOMETRY FOR PREPARATORY TWO FIRST TERM

PREPARED BY Mr. MAHMOUD



Sheet (1)
Medians of triangle

Definition

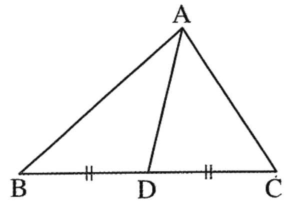
The median of a triangle is the line segment drawn from any vertex of this triangle to the midpoint of the opposite side of this vertex.

For example:

In the opposite figure :

If D is the midpoint of $\overline{BC}$

, then $\overline{AD}$ is a median of Δ ABC



Notice that :

Any triangle has three medians.

Theorem 1

The medians of a triangle are concurrent.

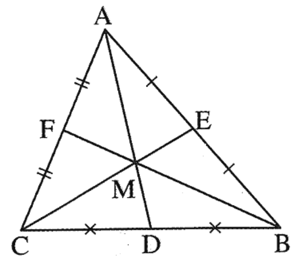
For example:

In the opposite figure :

$\overline{AD}$, $\overline{BF}$ and $\overline{CE}$ are the three medians of Δ ABC ,

and they are concurrent at M

(**i.e.** $\overline{AD} \cap \overline{BF} \cap \overline{CE} = \{M\}$)



Theorem 2

The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base.

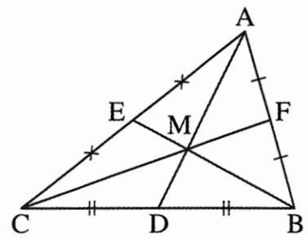
For example:

In the opposite figure :

In Δ ABC , M is the point of concurrence of its medians , then :

1 MD = $\frac{1}{2}$ AM If AM = 6 cm. , then MD = 3 cm.

2 CM = 2 FM If FM = 4 cm. , then CM = 8 cm.



Remark

The point of concurrence of the medians of the triangle divides each of them in the ratio of 2 : 1 from the vertex.

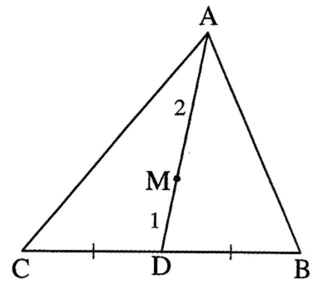
Fact

The point which divides the median in a triangle by the ratio of 1 : 2 from the base is the point of intersection of the medians of this triangle.

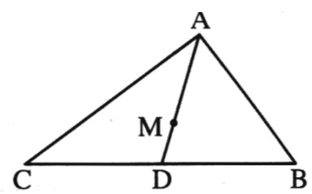
In the opposite figure :

If $\overline{AD}$ is a median in Δ ABC and M $\in \overline{AD}$ such that AM = 2 MD ,

then M is the point of intersection of the medians of Δ ABC

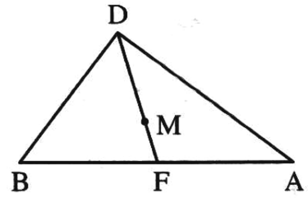


[1] Complete:

1	In Δ ABC : if the point X is the midpoint of $\overline{BC}$, then $\overline{AX}$ is called ....
2	The medians of the triangle are ....
3	The medians of the triangle intersect at ....
4	The point of intersection of the medians of a triangle divides each median in the ratio .... from the vertex.
5	The points of concurrence of the medians of the triangle divides each median in the ratio .... : .... from the base.
6	The point of intersection of the medians of the triangle divides each of them by the ratio 1 : 2 from ....
7	The point which divides the median of the triangle in the ratio 1 : 2 from the base is the point of ....
8	**In the opposite figure :** If M is the point of intersection of the medians of Δ ABC , then AM = ... AD 

9 **In the opposite figure :**

If : MF = 2 cm., then DF = ………

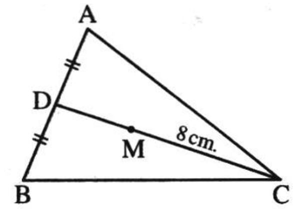


10 **In the opposite figure :**

In Δ ABC , M is the point of concurrence of the medians

, MC = 8 cm.

, then DM = ……… cm.



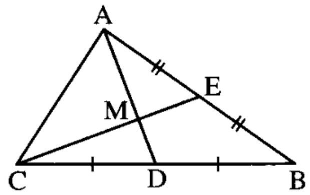
[2] Essay problems:

1 **In the opposite figure :**

E is the midpoint of $\overline{AB}$, D is the midpoint of $\overline{BC}$

$\overline{AD} \cap \overline{CE} = \{M\}$, MC = 5 cm. and MD = 2 cm.

Find : The length of each of $\overline{AD}$ and $\overline{ME}$.

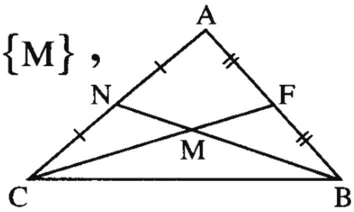


2 **In the opposite figure :**

F , N are midpoints of $\overline{AB}$, $\overline{AC}$ respectively , $\overline{BN} \cap \overline{CF} = \{M\}$,

if : AB = 8 cm. , AC = 10 cm. , BM = 4 cm. and CF = 9 cm.

Find : the perimeter of figure AFMN



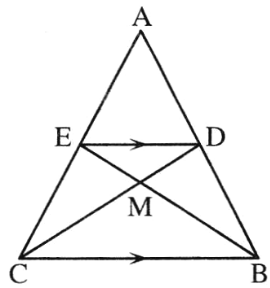
3 **In the opposite figure :**

ABC is a triangle in which $\overline{CD}$,

$\overline{BE}$ two medians intersects at M ,

if : DC = 9 cm. , BM = 4 cm. , BC = 8 cm.

Find : The perimeter of Δ MDE



Homework

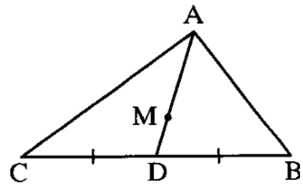
[1] Choose the correct answer:

1	The medians of the triangle intersect at ·········· point. (a) 1 (b) 2 (c) 3 (d) 4
2	The right-angled triangle has ·········· medians. (a) 0 (b) 1 (c) 2 (d) 3
3	The number of medians in the right-angled triangle = ··············· (a) 3 (b) 2 (c) 1 (d) 0
4	The point of intersection of the medians in the triangle divides each of them by the ratio ·········· from the vertex. (a) 1 : 3 (b) 3 : 1 (c) 2 : 1 (d) 1 : 2
5	The point of concurrence of the medians of the triangle divides each median in the ratio of ·········· from the base. (a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1
6	If $\overline{AD}$ is a median of triangle ABC , and M is the point of intersection of the medians , then AM = ·········· AD (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
7	$\overline{AD}$ is a median in Δ ABC , M is the point of intersection of its medians , then AM = ·········· MD (a) 2 (b) $\frac{1}{2}$ (c) 3 (d) $\frac{1}{3}$
8	If $\overline{XE}$ is a median in Δ XYZ , M is the point of intersection of its medians , then EM = ·········· XE (a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$
9	In Δ ABC : If AD = 6 cm. is a median and M is a point of concurrent , then MA = ·········· cm. (a) 6 cm. (b) 3 cm. (c) 2 cm. (d) 4 cm.
10	If $\overline{AD}$ is a median of Δ ABC , M is the point of intersection of its medians and AM = 6 cm. , then AD = ·········· (a) 12 cm. (b) 6 cm. (c) 18 cm. (d) 9 cm.

11 **In the opposite figure :**

$\overline{AD}$ is a median in Δ ABC , M is the point of intersection of the medians , MD = 2 cm. , then AD = ... cm.

(a) 2 (b) 4 (c) 6 (d) 8

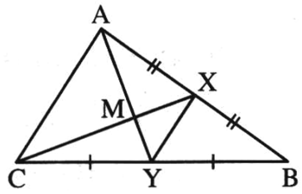


[2] Essay problems:

1 **In the opposite figure :**

ABC is a triangle , X bisects $\overline{AB}$, Y bisects $\overline{BC}$, XY = 5 cm. , $\overline{XC} \cap \overline{AY} = \{M\}$ where CM = 8 cm. , YM = 3 cm.

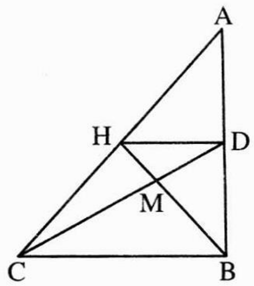
Find with proof the length of : $\overline{AC}$, $\overline{MX}$, $\overline{AM}$



2 **In the opposite figure :**

ABC is a triangle in which $\overline{CD}$, $\overline{BH}$ are medians intersect at M , MC = 6 cm. , BC = 8 cm. , MB = 4 cm.

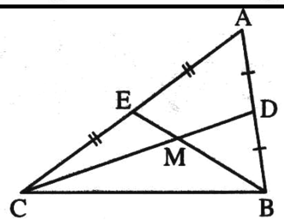
Find with proof : The perimeter of Δ MDH



3 **In the opposite figure :**

D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , $\overline{BE} \cap \overline{CD} = \{M\}$, If AB = 6 cm. , AC = 10 cm. , BM = 4 cm. and CD = 9 cm.

Find the perimeter of the figure : ADME



Sheet (2)

Medians of triangle (Follow)

Theorem 3

In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.

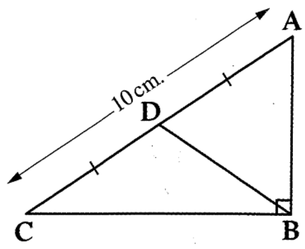
For example:

In the opposite figure :

ΔABC is a right-angled triangle at B ,

D is the midpoint of $\overline{AC}$ and AC = 10 cm. ,

then DB = 5 cm.



The converse of theorem 3

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.

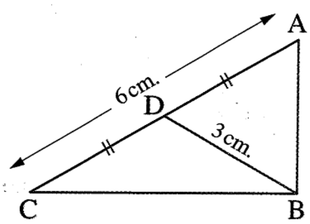
For example:

In the opposite figure :

If $\overline{BD}$ is a median in ΔABC ,

BD = 3 cm. and AC = 6 cm. ,

then m ($\angle$ ABC) = 90° "because BD = $\frac{1}{2}$ AC "



Corollary

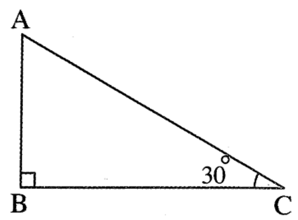
The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.

i.e.

In the opposite figure :

If ΔABC is right-angled at B and

m ($\angle$ C) = 30° , then AB = $\frac{1}{2}$ AC



For example:

If AC = 20 cm. , then AB = 10 cm.

Remark

The right-angled triangle whose measures of angles are 30° , 60° and 90° is called thirty and sixty triangle.

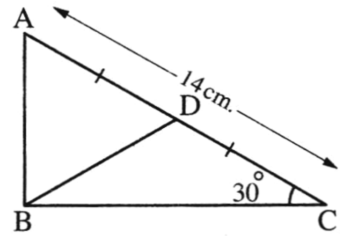
[1] Complete:

1	In the right-angled triangle the length of the median from the vertex of the right angle equal ……… the length of the hypotenuse.
2	In the right-angled triangle , the length of the median from the vertex of the right angle equals ………
3	If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex in length , then ………
4	The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ……… the length of the hypotenuse.
5	The length of side opposite to the angle whose measure = 30° in the right-angled triangle = ………
6	The length of the hypotenuse on the right-angled triangle equals ……… the length of a side opposite to the angle of measure 30°
7	In Δ LMN : If m (∠ L) = 30° , m (∠ N) = 60° , NM = 4 cm. , then LN = ……… cm.
8	If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , if $\overline{BD}$ is a median of triangle ABC , then BD = ……… cm.
9	In Δ ABC , m (∠ C) = 60° , m (∠ B) = 90° , AC = 8 cm. , then BC = …………… cm.
10	In Δ ABC if m (∠ A) = 30° and m (∠ B) = 90° , then BC = ……… AC
11	If ABC : Is a right-angled at B , AB = $\frac{1}{2}$ AC , then m (∠ C) = ………
12	If ABC is a right-angled triangle at B and AB = $\frac{1}{2}$ AC , then m (∠ A) = ………

13 ABC is a right-angled triangle at B , if AC = 2 BC , then m (∠ C) = ……………°

14 In the opposite figure :

The perimeter of Δ ABD = ……… cm.



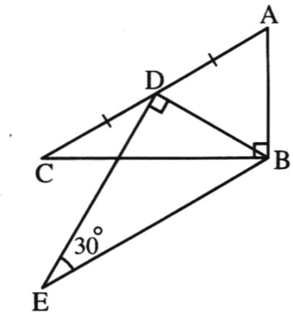
15 **In the opposite figure :**

D is the midpoint of $\overline{AC}$

, m (∠ E) = 30°

, AC = 10 cm.

Find the length of : $\overline{BE}$



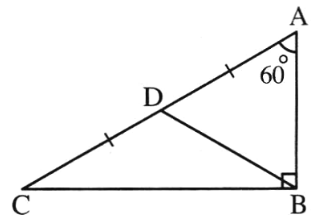
[2] Essay problems:

1 **In the opposite figure :** Δ ABC , AC = 8 cm. ,

m (∠ BAC) = 60° , m (∠ ABC) = 90° ,

D is the midpoint of $\overline{AC}$

Find : The perimeter of Δ ABD

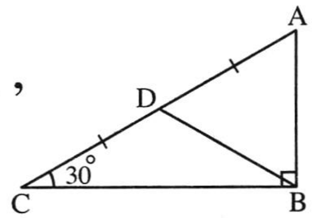


2 **In the opposite figure :**

m (∠ B) = 90° , m (∠ C) = 30° , $\overline{BD}$ is a median , AB = 4 cm. ,

Complete :

AC = ……… cm. , BD = ……… cm. , AD = ……… cm.



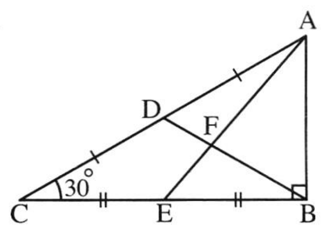
3 **In the opposite figure :**

Δ ABC in which m (∠ B) = 90° , AC = 10 cm. ,

m (∠ C) = 30° , EC = EB , AD = DC

Find with proof : 1 The perimeter of Δ ABD

2 The length of $\overline{DF}$



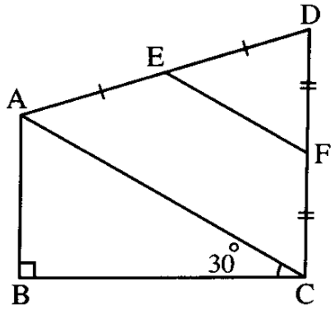
4 In the opposite figure :

m (∠ B) = 90° ,

m (∠ ACB) = 30° ,

E , F are midpoints of $\overline{AD}$, $\overline{DC}$

Prove that : AB = EF



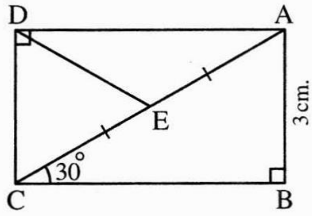
5 In the opposite figure :

m (∠ ABC) = m (∠ ADC) = 90° ,

m (∠ ACB) = 30° , and $\overline{DE}$ is a median of Δ ADC ,

If AB = 3 cm.

Find : The length of $\overline{DE}$



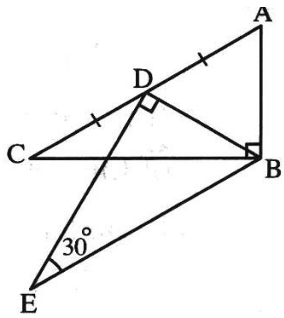
6 In the opposite figure :

m (∠ ABC) = m (∠ BDE) = 90°

, m (∠ E) = 30°

, D is the midpoint of $\overline{AC}$

Prove that : AC = BE



Homework

[1] Choose the correct answer:

1 The length of the hypotenous of the right-angled triangle = ... the length of the median which drawn from the vertex of the right-angle.

(a) half (b) twice (c) third (d) quarter

2 The length of the median drawn from the vertex of right angle in the right-angled triangle = ... the length of the hypotenuse of the triangle.

(a) 2 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

3 In the right-angled triangle , the length of the median from the vertex of the right angle equals .... the length of hypotenuse.

(a) half (b) twice (c) third (d) forth

4	In the right-angled triangle , the length of the median from the vertex of the right angle equal .... the length of the hypotenuse. (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 2
5	If Δ ABC is a right-angled at B , AB = 6 cm. , BC = 8 cm. , then the length of the medians drawn from B is .... cm. (a) 10 (b) 8 (c) 6 (d) 5
6	In Δ ABC which is right at B , if AC = 20 cm. , then the length of the median of the triangle drawn from B equals .... (a) 10 cm. (b) 8 cm. (c) 6 cm. (d) 5 cm.
7	In Δ ABC , m ($\angle$ B) = 90° , AC = 12 cm. and $\overline{BD}$ is a median in Δ ABC , then BD = ... cm. (a) 12 (b) 6 (c) 24 (d) 10
8	The length of the side opposite to the angle of measure 30° in the right-angled .... the length of the hypotenuse. (a) twice (b) half (c) square (d) equals
9	Triangle ABC : If m ($\angle$ A) = 30° , m ($\angle$ B) = 90° , then BC = ... (a) $\frac{1}{2}$ AB (b) $\frac{1}{2}$ AC (c) 2 AB (d) 2 AC
10	In Δ ABC if : m ($\angle$ B) = 90° and m ($\angle$ A) = 60° , then AC = ... AB (a) 2 (b) = (c) $\frac{1}{2}$ (d) $\frac{1}{3}$
11	Δ ABC : if m ($\angle$ A) = 30° and m ($\angle$ B) = 90° , then AC = ... (a) $\frac{1}{2}$ BC (b) 2 BC (c) 2 AB (d) BC
12	In Δ ABC : m ($\angle$ A) = 30° , m ($\angle$ B) = 90° , AC = 10 cm. , then BC = ... cm. (a) 20 (b) 15 (c) 10 (d) 5
13	In Δ XYZ , if m ($\angle$ Y) = 90° , m ($\angle$ X) = 30° and XZ = 20 cm. , then ZY = ... cm. (a) 5 (b) 8 (c) 20 (d) 10

14 In the rectangle ACBD , if AC = 10 cm. , then BD = ...

(a) 5 (b) 10 (c) 15 (d) 20

[2] Complete:

1 In the right-angled triangle , the length of the median from the vertex of the right angle equals ....

2 If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex in length , then ....

3 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

4 The length of side opposite to the angle whose measure = 30° in the right-angled triangle = ...

5 The length of the hypotenuse on the right-angled triangle equals .... the length of a side opposite to the angle of measure 30°

6 In Δ LMN : If m (∠ L) = 30° , m (∠ N) = 60° , NM = 4 cm. , then LN = ... cm.

7 If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , if $\overline{BD}$ is a median of triangle ABC , then BD = ... cm.

8 In Δ ABC , m (∠ C) = 60° , m (∠ B) = 90° , AC = 8 cm. , then BC = ... cm.

9 In Δ ABC if m (∠ A) = 30° and m (∠ B) = 90° , then BC = ... AC

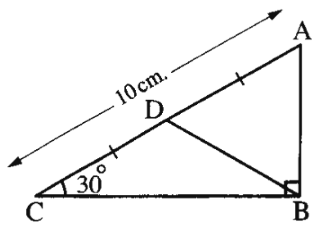
[3] Essay problems:

1 **In the opposite figure :**

m (∠ B) = 90° and m (∠ C) = 30° ,

AC = 10 cm.

Find : the lengths of $\overline{AB}$ and $\overline{BD}$



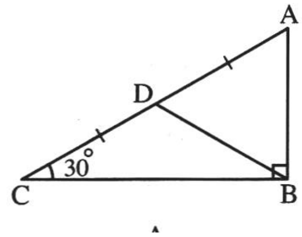
2

In the opposite figure :

m (∠ C) = 30°

Prove that :

AB = BD



3

In the opposite figure :

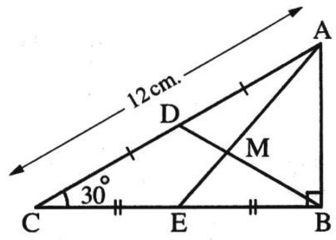
In Δ ABC : m (∠ B) = 90° , m (∠ C) = 30°

, D is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

, AC = 12 cm.

(1) **Find length of :** $\overline{BM}$

(2) **Find the perimeter of :** Δ ABC



4

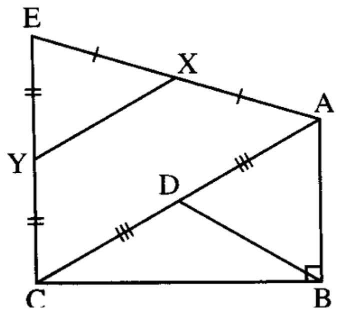
In the opposite figure :

X , Y , D are the midpoints of

$\overline{EA}$, $\overline{EC}$ and $\overline{AC}$ respectively ,

m (∠ ABC) = 90°

Prove that : BD = YX



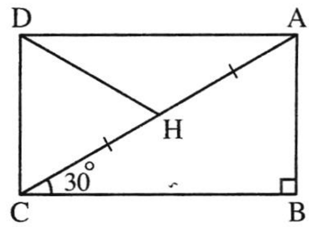
5

In the opposite figure :

m (∠ B) = 90° , m (∠ ACB) = 30° ,

AB = DH where H is midpoint of $\overline{AC}$

Prove that : m (∠ ADC) = 90°



6

In the opposite figure :

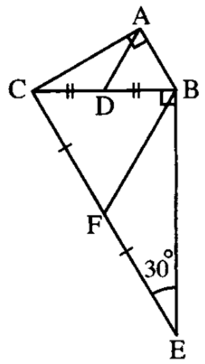
m (∠ BAC) = m (∠ CBE) = 90° ,

m (∠ BEC) = 30° ,

D and F are the midpoints of $\overline{BC}$

and $\overline{CE}$ respectively.

Prove that : AD = $\frac{1}{2}$ BF

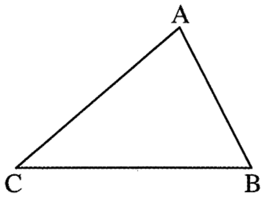


Sheet (3)

The isosceles triangle

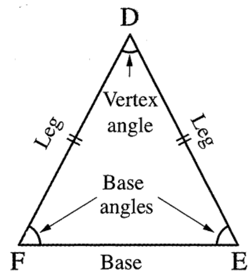
Triangles are classified according to the lengths of their sides into three types which are :

1 Scalene triangle.



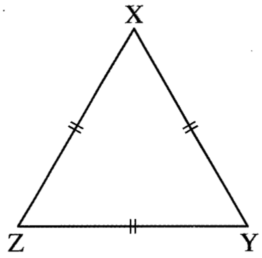
$AB \neq BC \neq CA$

2 Isosceles triangle.
(two sides are congruent).



$DE = DF$

3 Equilateral triangle.
(three sides are congruent).



$XY = YZ = ZX$

And in the following we will study the relations between the angles in the isosceles triangle and the equilateral triangle.

The isosceles triangle theorem

Theorem 1

The base angles of the isosceles triangle are congruent.

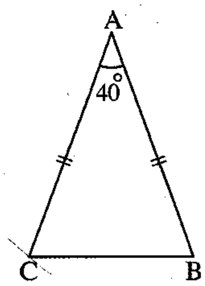
For example:

In the opposite figure :

If ABC is a triangle in which :

$AB = AC$, $m(\angle A) = 40°$,

then $m(\angle B) = m(\angle C) = \dfrac{180° - 40°}{2} = 70°$



Remarks

1 Both of the base angles in the isosceles triangle are acute.

2 The vertex angle in the isosceles triangle may be acute , right or obtuse angle.

Corollary

If the triangle is equilateral , then it is equiangular where each angle measure is 60°

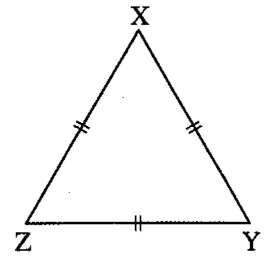
For example:

In the opposite figure :

If XYZ is a triangle in which

XY = YZ = ZX ,

then m (∠ X) = m (∠ Y) = m (∠ Z) = 60°



[1] Complete:

1	The two base angles in an isosceles triangle are ……….
2	Δ ABC , AB = AC , m (∠ C) = 70° , then m (∠ A) = ……….
3	In the Δ ABC : AB = AC , m (∠ A) = 70° , then m (∠ C) = ……….°
4	The Δ ABC is an isosceles and right-angled triangle if m (∠ B) = 90° , then m (∠ A) = m (∠ C) = ……………°
5	In Δ ABC , if AB = AC and m (∠ A) = 80° , then m (∠ B) = m (∠ ……………) = ……..°
6	In Δ ABC : if AB = AC , m (∠ B) = 60° , then the triangle is an ……….
7	In Δ ABC : If AB = AC and m (∠ A) = 2 m (∠ C) , then m (∠ B) = ………. °
8	The length of side opposite to the angle whose measure = 30° in the right-angled triangle = ……….
9	The length of the hypotenuse on the right-angled triangle equals ………. the length of a side opposite to the angle of measure 30°
10	In Δ LMN : If m (∠ L) = 30° , m (∠ N) = 60° , NM = 4 cm. , then LN = …….. cm.
11	If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , if $\overline{BD}$ is a median of triangle ABC , then BD = ………. cm.
12	In Δ ABC , m (∠ C) = 60° , m (∠ B) = 90° , AC = 8 cm. , then BC = …………… cm.

[2] Essay problems:

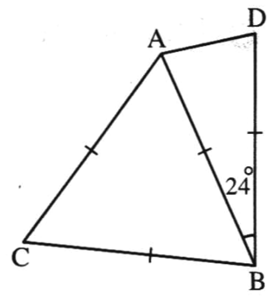
1 **In the opposite figure :**

ACBD is a quadrilateral in which :

AB = BC = CA = BD

, m (∠ ABD) = 24°

Find : m (∠ CAD)

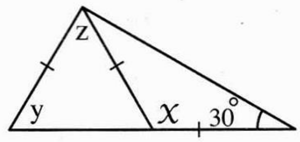


2 **In the opposite figure complete :**

x = ……… ° ,

y = ……… ° ,

z = ……… °

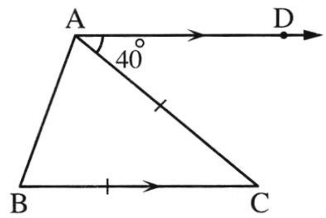


3 **In the opposite figure :**

ABC is a triangle ,

AC = BC , $\overrightarrow{AD}$ // $\overline{BC}$, m (∠ DAC) = 40°

Find : The measure of angles in the Δ ABC

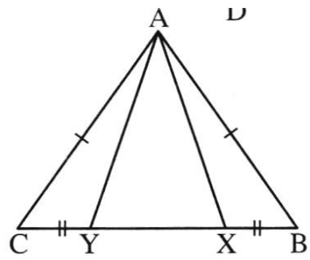


4 **In the opposite figure :**

In Δ ABC , AB = AC ,

BX = CY

Prove that : AX = AY



Homework

[1] Choose the correct answer:

1 In any isosceles triangle , the type of the base angles is ………

(a) acute. (b) right. (c) obtuse. (d) reflex.

2 The base angles of the isosceles triangle are ……………

(a) congruent. (b) alternate. (c) corresponding. (d) supplementary.

3 In Δ ABC : AB = AC , m (∠ B) = 50° , then m (∠ A) = ……………°

(a) 65 (b) 80 (c) 50 (d) 100

4	If measure of one of the two base angles of the isosceles triangle equals 40° then the measure of the vertex angle = ……………° (a) 40 (b) 100 (c) 80 (d) 50
5	An isosceles triangle , one of its base angles has measure 50° , then the measure of the vertex angle = ……… (a) 50° (b) 60° (c) 70° (d) 80°
6	In the isosceles triangle , if the measure of one of the two base angle is 70° , then the measure of its vertex angle is ……… (a) 70° (b) 110° (c) 20° (d) 40°
7	The measure of one angle of the two base angles of the isosceles = 75° , then the measure of the vertex angle = ……… (a) 50° (b) 75° (c) 30° (d) 105°
8	In a triangle ABC : If AB = AC and m (∠ A) = 40° , then m (∠ C) = ……… (a) 40° (b) 70° (c) 140° (d) 50°
9	In Δ ABC , AB = AC , m (∠ A) = 50° , then m (∠ B) = ……… (a) 50° (b) 65° (c) 130° (d) 100°
10	If the measure of an angle of the isosceles triangle is 100° , then the measure of one of the other angles = ……… (a) 50° (b) 80° (c) 40° (d) 100°
11	If the measure of an angle of the isosceles triangles is 120° , then the measure of one of the other angles = ……… (a) 60° (b) 30° (c) 40° (d) 45°
12	The triangle whose sides lengths are 2 cm. , $(X + 1)$ cm and 5 cm. becomes an isosceles triangle when X = ……… cm. (a) 1 (b) 2 (c) 3 (d) 4
13	Triangle whose sides lengths are 2 cm. , $(X - 2)$ cm. , 5 cm. becomes isosceles triangle when X = ……… cm. (a) 3 (b) 4 (c) 5 (d) 7

14 The triangle whose sides lengths are 3 cm. , $(X + 5)$ and 9 becomes an isosceles if $X =$ ·········· cm.

(a) 3 (b) 4 (c) 5 (d) 6

15 ABC is a triangle in which AB = AC and m (∠ A) = 110° , then m (∠ B) = ··········

(a) 70° (b) 55° (c) 35° (d) 110°

16 Δ XYZ is an isosceles triangle in which m (∠ X) = 100° , then m (∠ Y) = ··········°

(a) 100 (b) 80 (c) 60 (d) 40

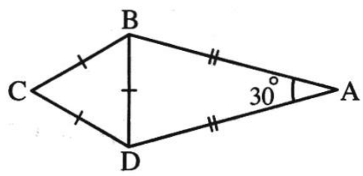
[2] Essay problems:

1 **In the opposite figure :**

AB = AD , m (∠ A) = 30° ,

CB = BD = CD

Find : m (∠ CBA)

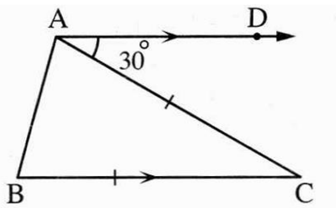


2 **In the opposite figure :**

ABC is a triangle in which : AC = BC ,

$\overline{AD} // \overline{BC}$, m (∠ DAC) = 30°

Find : m (∠ ABC)

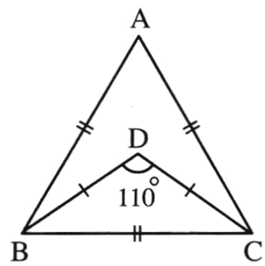


3 **In the opposite figure :**

ABC is an equilateral triangle ,

DB = DC , m (∠ D) = 110°

Find with proof : m (∠ DBC) and m (∠ DBA)

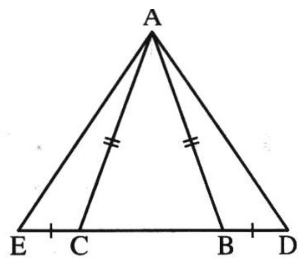


4 **In the opposite figure :**

ADE is a triangle , B ∈ $\overline{DE}$, C ∈ $\overline{DE}$

, BD = CE , AB = AC

Prove that : AD = AE



Sheet (4)

The converse of the isosceles triangle theorem

Sheet (4)
The converse of the isosceles triangle theorem

Theorem 2

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.

Remark

The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.

[1] Complete:

1 If angles of any triangle are equal in measures , then the triangle is ………

2 If the angles of a triangle are congruent , then the triangle is ……………

3 The measure of the exterior angle of equilateral triangle = ………°

4 If the measure of one of the angles of the right-angled triangle is 45° , then the triangle is ……………

5 In an isosceles triangle , if any angle has a measure of 60° , the triangle is ………

6 In Δ ABC if : $\overline{AB} \perp \overline{BC}$ and AB = BC , then m ($\angle$ A) = ………°

[2] Essay problems:

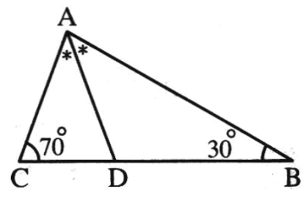
1 **In the opposite figure :**

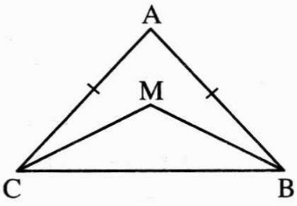
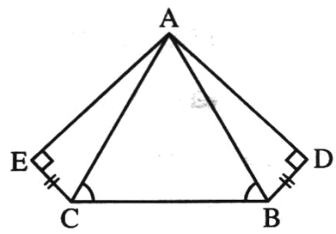
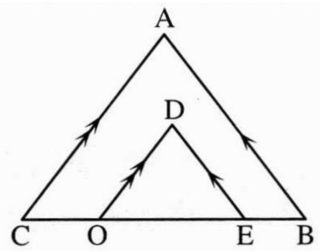
$\overrightarrow{AD}$ bisects $\angle$ BAC

, m ($\angle$ B) = 30°

, m ($\angle$ C) = 70°

Prove that : Δ ADC is isosceles triangle.



2	ABC is a triangle in which : m (∠ A) = 50° and m (∠ C) = 80° **Prove that :** this triangle ABC is an isosceles triangle.
3	**In the opposite figure :** AB = AC , $\overrightarrow{BM}$ and $\overrightarrow{CM}$ bisect the angles (∠ B) , (∠ C) **Prove that :** MB = MC 
4	**In the opposite figure :** BD = CE , m (∠ ABC) = m (∠ ACB) , m (∠ D) = m (∠ E) = 90° **Prove that :** m (∠ DAB) = m (∠ CAE) 
5	**In the opposite figure :** AB = AC , $\overline{DE}$ // $\overline{AB}$ and $\overline{AC}$ // $\overline{DO}$ **Prove that :** ❶ DE = DO ❷ m (∠ A) = m (∠ D) 

Homework

[1] Choose the correct answer:

1	The measure of exterior angle of an equilateral triangle = ………			
	(a) 30°	(b) 60°	(c) 120°	(d) 180°
2	In Δ XYZ : if XY = XZ , then the exterior angle at the vertex Z is ………			
	(a) acute.	(b) obtuse.	(c) right.	(d) reflex.
3	In Δ ABC : if AB = AC and m (∠ A) = 60° , if its perimeter is 18 cm. , then BC = ……… cm.			
	(a) 18	(b) 6	(c) 3	(d) 60

4 | Δ ABC , AB = AC , D is the midpoint of $\overline{BC}$, then $\overline{AD}$ is ………

(a) median. (b) altitude.

(c) bisector of the vertex angle. (d) all the previous.

[2] Essay problems:

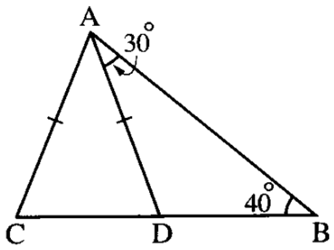
1 **In the opposite figure :**

AD = AC

, m (∠ DAB) = 30°

, m (∠ ABD) = 40°

Prove that : AB = CB

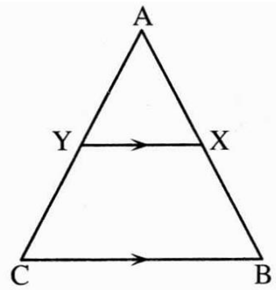


2 **In the opposite figure :**

ABC is a triangle in which AB = AC , X ∈ $\overline{AB}$,

Y ∈ $\overline{AC}$ and $\overline{XY}$ // $\overline{BC}$

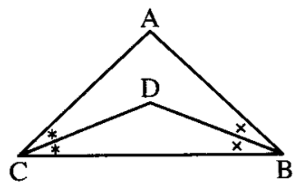
Prove that : the triangle AXY is isosceles triangle.



3 **In the opposite figure :**

AB = AC , $\overrightarrow{BD}$ bisects ∠ B and $\overrightarrow{CD}$ bisects ∠ C

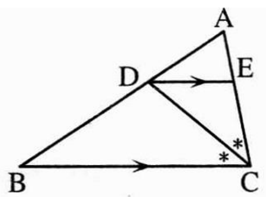
Prove that : Δ DBC is an isosceles triangle



4 **In the opposite figure :**

$\overrightarrow{CD}$ bisects ∠ ACB , $\overline{DE}$ // $\overline{CB}$

Prove that : Δ ECD is an isosceles triangle.



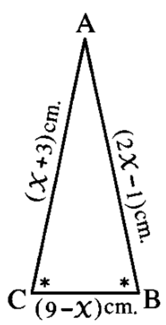
5 **In the opposite figure :**

m (∠ B) = m (∠ C) , AB = (2 X – 1) cm.

AC = (X + 3) cm.

, BC = (9 – X) cm.

Find with proof the perimeter of Δ ABC



Sheet (5)

Corollaries of the isosceles triangle theorems

Corollary 1

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

In the opposite figure :

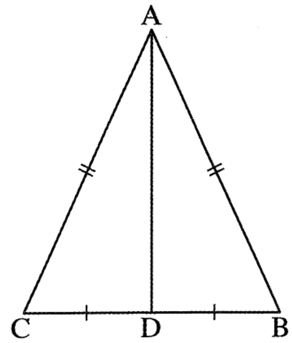
ABC is a triangle in which AB = AC and

$\overline{AD}$ is a median , then :

1 $\overrightarrow{AD}$ bisects $\angle$ BAC

i.e. m ($\angle$ BAD) = m ($\angle$ CAD)

2 $\overline{AD} \perp \overline{BC}$



Corollary 2

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

In the opposite figure :

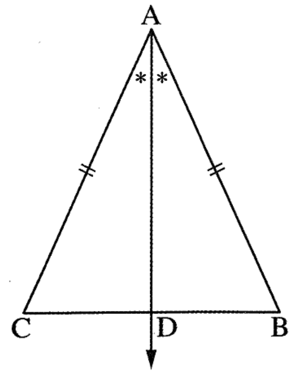
ABC is a triangle in which AB = AC and

$\overrightarrow{AD}$ bisects $\angle$ BAC , then :

1 D is the midpoint of $\overline{BC}$

i.e. BD = CD

2 $\overline{AD} \perp \overline{BC}$



Corollary 3

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.

In the opposite figure :

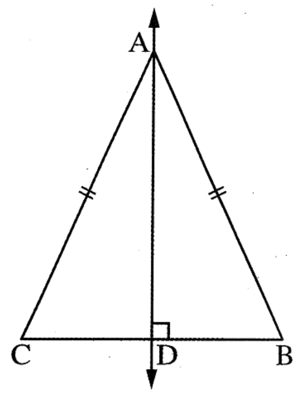
ABC is a triangle in which AB = AC and

$\overleftrightarrow{AD} \perp \overline{BC}$, then :

1 D is the midpoint of $\overline{BC}$

i.e. BD = CD

2 m ($\angle$ BAD) = m ($\angle$ CAD)



Notice that :

The previous three corollaries can be proved using the congruence of Δ ABD and Δ ACD

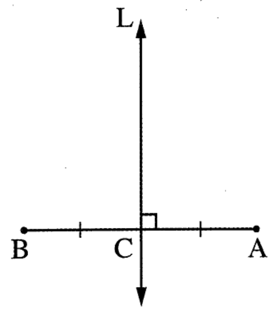
Axis of symmetry of a line segment

Definition

The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment , in brief it is known as the axis of a line segment.

In the opposite figure :

If the straight line L $\perp$ $\overline{AB}$ and C $\in$ the straight line L where C is the midpoint of $\overline{AB}$, then the straight line L is called the axis of $\overline{AB}$



Property

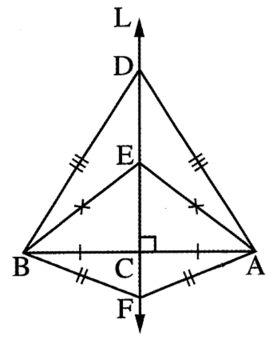
Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).

In the opposite figure :

If the straight line L is the axis of $\overline{AB}$,

D $\in$ L , E $\in$ L and F $\in$ L , then

DA = DB , EA = EB and FA = FB

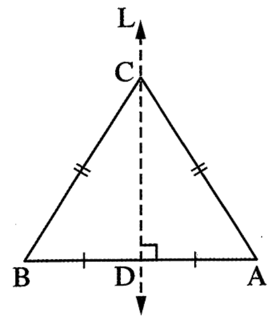


The converse of the previous property is true

i.e. If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

In the opposite figure :

If C is a point such that CA = CB , then the point C lies on the axis of $\overline{AB}$



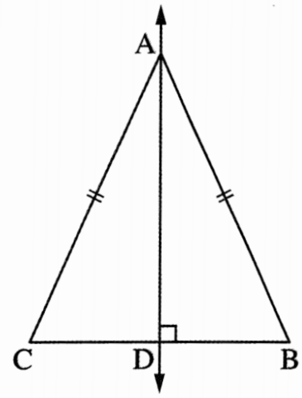
Axis of symmetry of the isosceles triangle

The isosceles triangle has one axis of symmetry.

It is the straight line drawn from the vertex angle perpendicular to its base.

For example:

If ABC is an isosceles triangle where $AB = AC$ and $\overleftrightarrow{AD} \perp \overline{BC}$, then $\overleftrightarrow{AD}$ is called the axis of symmetry of the isosceles triangle ABC

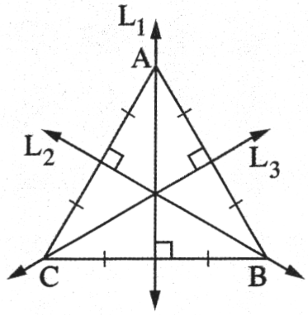


Remarks

1 The equilateral triangle has three axes of symmetry , they are the three perpendiculars drawn from its vertices to the opposite sides.

In the opposite figure :

The straight lines L_1 , L_2 and L_3 are the axes of symmetry of the equilateral triangle ABC



2 The scalene triangle has no axes of symmetry.

[1] Complete:

1	The ray drawn from the vertex of the isosceles triangle passing through the midpoint of the base is ……………
2	The median of an isosceles triangle drawn from the vertex bisects ……… and is perpendicular to ………
3	The bisector of the vertex angle of an isosceles triangle ……… and ………
4	In ΔXYZ : If $XY = XZ$, $\overrightarrow{XL} \perp \overline{YZ}$, then $\overleftrightarrow{XL}$ bisects each of ……… and ………
5	The straight line perpendicular to the midpoint of a line segment is called ………
6	In the isosceles triangle if the measure of any angle is 60° , then the number of axis of symmetry ………
7	The number of axes of symmetry of the isosceles triangle equal ………
8	The number of symmetrical line in an scalene triangle = ………

9	The number of the axes of symmetry in an equilateral triangle = ...
10	The number of axes of symmetry of the triangle in which the measures of two angles are 50° , 70° = ...
11	In Δ ABC : If AB = AC , then the point A lies on the axis of symmetry of ....
12	If D is the midpoint of $\overline{AB}$ and $\overleftrightarrow{CD} \perp \overline{AB}$, then CA = ...
13	The axis of symmetry of the line segment is the straight line which ....
14	Any point on the axis symmetry of a line segment is at two equal distance from ....
15	If the point A $\in$ the axis of symmetry of $\overline{BC}$, then AB = ...
16	The axis of symmetry of isosceles triangle is ....

[2] Essay problems:

1 **In the opposite figure :**

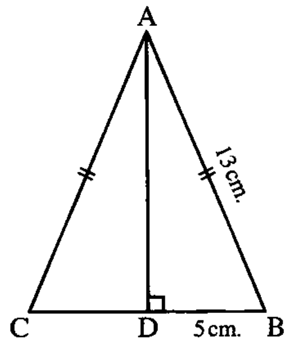
In Δ ABC , AB = AC ,

$\overline{AD} \perp \overline{BC}$,

AB = 13 cm. and BD = 5 cm.

Find : ❶ The length of $\overline{BC}$

❷ The area of Δ ABC

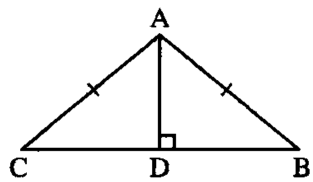


2 **In the opposite figure :**

ABC is a triangle in which : AB = AC , $\overline{AD} \perp \overline{BC}$

m ($\angle$ BAC) = 100° and BD = 3 cm.

Find : (1) m ($\angle$ BAD) (2) The length of $\overline{CB}$



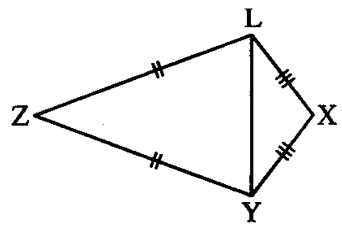
3 **In the opposite figure :**

XL = XY , ZL = ZY ,

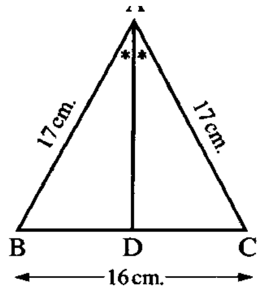
M is the midpoint of $\overline{LY}$

Prove that :

X , M , Z are on the same straight line.



4 **In the opposite figure :**
$\overrightarrow{AD}$ bisects $\angle$ BAC ,
AB = AC = 17 cm. ,
and BC = 16 cm.
Prove that : m ($\angle$ ADB) = 90° ,
then find the length of : $\overline{AD}$ and the area of Δ ABC



Homework

[1] Choose the correct answer:

1	The axis of symmetry of a line segment is the straight line which is .... (a) Perpendicular to it. (b) its bisector. (c) parallel to it. (d) the perpendicular bisector.
2	If A $\in$ the axis of symmetry of $\overline{BC}$, then $\overline{AB}$ .... $\overline{AC}$ (a) $\perp$ (b) $\equiv$ (c) // (d) =
3	If A lies on the axis of symmetry of $\overline{XY}$ then AX .... AY (a) // (b) $\perp$ (c) = (d) $\neq$
4	The number of axis of symmetry in the scalene triangle is .... (a) 1 (b) zero (c) 3 (d) 4
5	The number of axes of symmetry in the isosceles triangle is .... (a) 1 (b) 2 (c) 3 (d) zero
6	The isosceles triangle has .... axis (axes) of symmetry. (a) no (b) two (c) only one (d) three
7	The number of axes of symmetry in the equilateral triangle is .... (a) 0 (b) 2 (c) 3 (d) 1
8	The equilateral triangle has .... axes of symmetry. (a) one (b) two (c) three (d) otherwise

9	The triangle which has no axes of symmetry is ……… triangles. (a) scalene (b) isosceles (c) equilateral (d) otherwise
10	If Δ ABC has one axes of symmetry and m ($\angle$ ABC) = 140° , then m ($\angle$ A) = ……… (a) 30° (b) 20° (c) 40° (d) 60°
11	The triangle which has three axes of symmetry is ……… triangle. (a) scalene (b) isosceles (c) right-angled (d) equilateral
12	Δ ABC in which m ($\angle$ A) = m ($\angle$ B) = 65° , then it has …………… axis (axes) of symmetry. (a) 1 (b) 2 (c) 3 (d) zero
13	In Δ ABC if : m ($\angle$ A) = 40° and m ($\angle$ B) = 70° , then Δ ABC has …………… axis (axes) of symmetry. (a) 3 (b) 1 (c) 2 (d) zero
14	The quadrilateral ABCD in which $\overleftrightarrow{BD}$ is an axis of symmetry of $\overline{AC}$ may by ……… (a) a rhombus (b) a rectangle (c) a parallelogram (d) a trapezium

[2] Essay problems:

1 **In the opposite figure :**

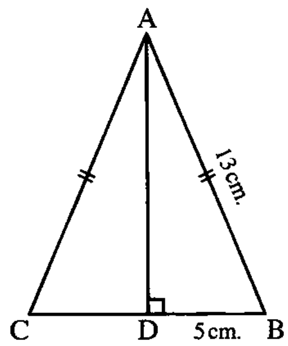
In Δ ABC , AB = AC ,

$\overline{AD} \perp \overline{BC}$,

AB = 13 cm. and BD = 5 cm.

Find : ❶ The length of $\overline{BC}$

❷ The area of Δ ABC

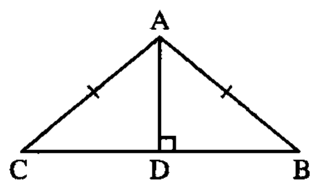


2 **In the opposite figure :**

ABC is a triangle in which : AB = AC , $\overline{AD} \perp \overline{BC}$

m ($\angle$ BAC) = 100° and BD = 3 cm.

Find : (1) m ($\angle$ BAD) (2) The length of $\overline{CB}$



3

In the opposite figure :

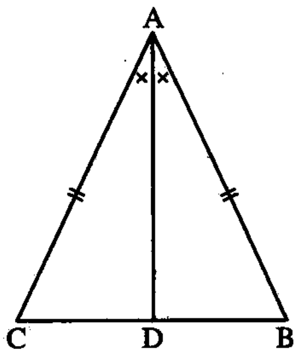
In ΔABC :

$AB = AC$, $\overrightarrow{AD}$ bisects $\angle BAC$

and $BD = 3$ cm.

Prove that : $\overline{AD} \perp \overline{BC}$

, then find the length of : $\overline{CB}$



4

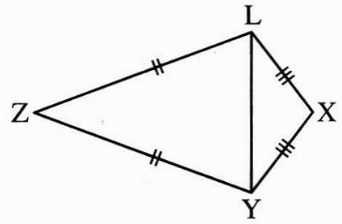
In the opposite figure :

$XL = XY$, $ZL = ZY$,

M is the midpoint of $\overline{LY}$

Prove that :

X , M , Z are on the same straight line.



Sheet (6)

Comparing the measure of angles in a triangle

Sheet (6)
Comparing the measure of angles in a triangle

Axioms of inequality relation

For any four numbers a , b , c and d :

1. If $a > b$, then $a + c > b + c$
2. If $a > b$, then $a - c > b - c$
3. If $a > b$, $c > 0$, then $a\,c > b\,c$
4. If $a > b$, $b > c$, then $a > c$
5. If $a > b$, $c > d$, then $a + c > b + d$

Remember that :

The measure of any exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

Theorem

In a triangle , if two sides have unequal lengths , then the longer is opposite to the angle of the greater measure.

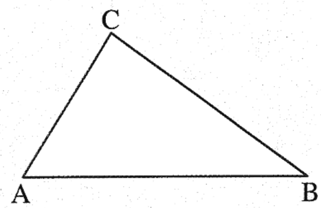
Remark

The greatest angle in measure of the triangle is opposite to the longest side of the triangle and its measure is greater than 60°.and the smallest angle in measure of the triangle is opposite to the shortest side of the triangle and its measure is less than 60°

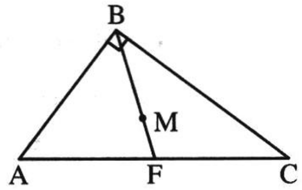
i.e. In ΔABC :

If $AB > BC > AC$, then $m(\angle C) > m(\angle A) > m(\angle B)$

, $m(\angle C) > 60°$ and $m(\angle B) < 60°$

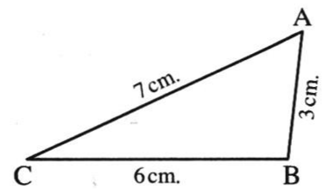


[1] Complete:

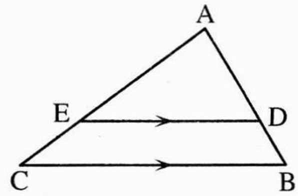
1	The length of two sides in the triangle are not equal , then the greatest side in length is opposite to ....
2	In a triangle , if two sides have unequal lengths , the longer is opposite to the angle of the ....
3	In triangle ABC , if BC > AB , then m (∠ A) .... m (∠ C)
4	In Δ ABC : AB > AC , then m (∠ C) .... m (∠ B)
5	In Δ ABC , if AB > BC > AC , then the smallest angle in measure of it is angle ....
6	In Δ ABC : if the point X is the midpoint of $\overline{BC}$, then $\overline{AX}$ is called ....
7	The medians of the triangle are ....
8	The medians of the triangle intersect at ....
9	The point of intersection of the medians of a triangle divides each median in the ratio .... from the vertex.
10	The points of concurrence of the medians of the triangle divides each median in the ratio .... : .... from the base.
11	The point of intersection of the medians of the triangle divides each of them by the ratio 1 : 2 from ....
12	The point which divides the median of the triangle in the ratio 1 : 2 from the base is the point of ....
13	**In the opposite figure :** If M is intersection point of medians and m (∠ B) = 90° , MF = 1.5 cm. , then the length of $\overline{AC}$ = ... 

[2] Essay problems:

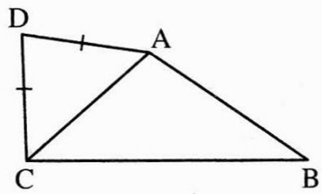
1 **In the opposite figure :**
Arrange the angles of Δ ABC descendingly due to their measures



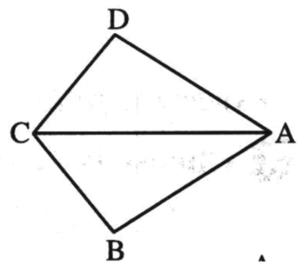
2 **In the opposite figure :**
$\overline{ED} \parallel \overline{BC}$,
$AC > AB$
Prove that : $AE > AD$



3 **In the opposite figure :**
ABCD is a quadrilateral in which : $AD = DC$,
$BC > AB$
Prove that : $m(\angle BAD) > m(\angle BCD)$



4 **In the opposite figure :**
$AD > DC$
and $AB > BC$
Prove that : $m(\angle BCD) > m(\angle BAD)$



Homework

[1] Choose the correct answer:

1 In Δ ABC , $AB > AC$, then $m(\angle C)$ .... $m(\angle B)$
(a) $<$ (b) $>$ (c) $=$ (d) $\leq$

2 In Δ ABC if $AB > AC$, then $m(\angle B)$ .... $m(\angle C)$
(a) $>$ (b) $<$ (c) $=$ (d) $\geq$

3 In Δ ABC , $AB > AC$, $m(\angle C) = 70°$, then $m(\angle B)$ may be ....
(a) 70° (b) 50° (c) 80° (d) 75°

4 In Δ ABC : If BC > AB , then m ($\angle$ A) .... m ($\angle$ C)

(a) = (b) < (c) ≤ (d) >

5 In the triangle XYZ , if XY > ZX , then m ($\angle$ Y) .... m ($\angle$ Z)

(a) > (b) < (c) = (d) ≥

6 In Δ ABC : AB = AC , m ($\angle$ B) = 65° , then : AC .... BC

(a) < (b) > (c) = (d) ≤

7 In Δ ABC : If AB = 9 cm. , BC = 6 cm. , AC = 7 cm. , then the smallest angle is ....

(a) $\angle$ BAC (b) $\angle$ ABC (c) $\angle$ ACB (d) $\angle$ BCA

8 The medians of the triangle intersect at .... point.

(a) 1 (b) 2 (c) 3 (d) 4

9 The right-angled triangle has .... medians.

(a) 0 (b) 1 (c) 2 (d) 3

10 The point of concurrence of the medians of the triangle divides each median in the ratio of .... from the base.

(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1

11 In Δ ABC : AB = AC , m ($\angle$ B) = 65° , then : AC .... BC

(a) < (b) > (c) = (d) ≤

12 In Δ ABC : If AB = 9 cm. , BC = 6 cm. , AC = 7 cm. , then the smallest angle is ....

(a) $\angle$ BAC (b) $\angle$ ABC (c) $\angle$ ACB (d) $\angle$ BCA

13 If $\overline{AD}$ is a median of triangle ABC , and M is the point of intersection of the medians , then AM = ... AD

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

14 $\overline{AD}$ is a median in Δ ABC , M is the point of intersection of its medians , then AM = ... MD

(a) 2 (b) $\frac{1}{2}$ (c) 3 (d) $\frac{1}{3}$

15 If $\overline{XE}$ is a median in Δ XYZ , M is the point of intersection of its medians , then EM = ·········· XE

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

16 In Δ ABC : If AD = 6 cm. is a median and M is a point of concurrent , then MA = ·········· cm.

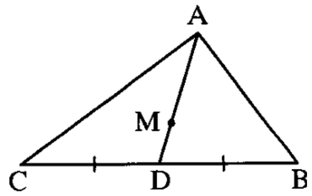
(a) 6 cm. (b) 3 cm. (c) 2 cm. (d) 4 cm.

17 If $\overline{AD}$ is a median of Δ ABC , M is the point of intersection of its medians and AM = 6 cm. , then AD = ··········

(a) 12 cm. (b) 6 cm. (c) 18 cm. (d) 9 cm.

18 **In the opposite figure :**

$\overline{AD}$ is a median in Δ ABC , M is the point of intersection of the medians , MD = 2 cm. , then AD = ·············· cm.



(a) 2 (b) 4 (c) 6 (d) 8

19 The number of medians in the right-angled triangle = ···············

(a) 3 (b) 2 (c) 1 (d) 0

20 The point of intersection of the medians in the triangle divides each of them by the ratio ·········· from the vertex.

(a) 1 : 3 (b) 3 : 1 (c) 2 : 1 (d) 1 : 2

[2] Essay problems:

1 In Δ ABC if : AB = 14 cm. , BC = 6 cm. and AC = 10 cm. Arrange the angles of Δ ABC ascendingly due to their measures.

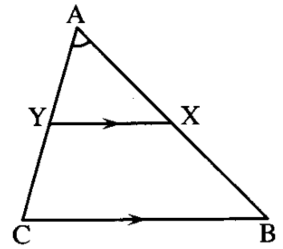
2 Δ XYZ in which , XY = 8 cm. , YZ = 10 cm. and ZX = 7 cm. ordered the measures of its angles ascending.

3 **In the opposite figure :**

ABC is a triangle ,

AB > AC , $\overline{XY}$ // $\overline{BC}$

Prove that : m ($\angle$ AYX) > m ($\angle$ AXY)



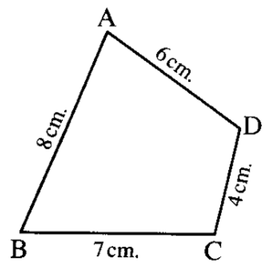
4 **In the opposite figure :**

AB = 8 cm. ,

BC = 7 cm. ,

CD = 4 cm. , AD = 6 cm.

Prove that : m ($\angle$ BCD) > m ($\angle$ BAD)

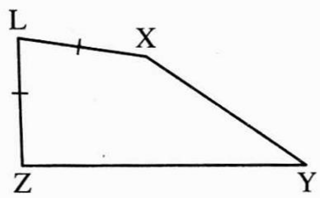


5 **In the opposite figure :**

XYZL is a quadrilateral ,

XL = LZ , YZ > XY

Prove that : m ($\angle$ LXY) > m ($\angle$ LZY)



6 **In the opposite figure :**

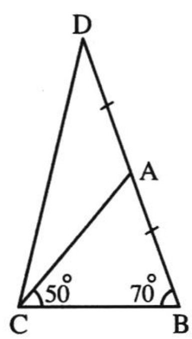
A is the midpoint of $\overline{BD}$

, m ($\angle$ ABC) = 70°

, m ($\angle$ ACB) = 50°

Prove that :

m ($\angle$ D) > m ($\angle$ DCA)



Sheet (7)

Comparing the lengths of sides in a triangle

Sheet (7)
Comparing the lengths of sides in a triangle

Theorem

In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

Corollaries

Corollary 1

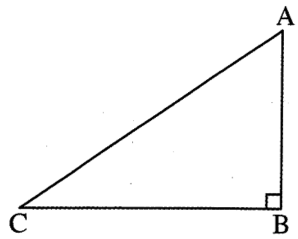
In the right-angled triangle , the hypotenuse is the longest side.

In the opposite figure :

If Δ ABC is right-angled at B , then m ($\angle$ B) > m ($\angle$ A) ,

m ($\angle$ B) > m ($\angle$ C) because $\angle$ B is a right angle and each of $\angle$ A and $\angle$ C is acute , so we find that :

AC > BC and AC > AB (according to the previous theorem).



Notice that :

In the obtuse-angled triangle, the side opposite to the obtuse angle is the longest side in the triangle.

Corollary 2

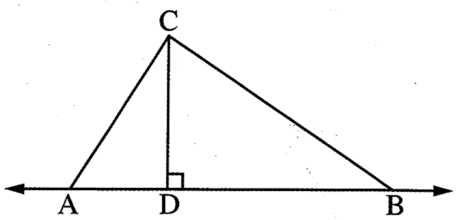
The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given straight line.

In the opposite figure :

If C $\notin \overleftrightarrow{AB}$ and D $\in \overleftrightarrow{AB}$ such that $\overline{CD} \perp \overleftrightarrow{AB}$,

then $\overline{CB}$ is the hypotenuse in Δ CBD

which is right-angled at D ,



$\overline{CA}$ is the hypotenuse in Δ CDA which is right-angled at D and so on ...

According to corollary 1 , we find that CB > CD , CA > CD and so on ...

i.e. CD < CB and CD < CA

Definition

The distance between any point and a given straight line is the length of the perpendicular line segment drawn from this point to the given line.

[1] Complete:

1	In the right-angled triangle the longest side in it called ………
2	The longest side length in the right-angled triangle is ………
3	If XYZ is a right-angled triangle at Y , then the longest side is ………
4	In a triangle if two angles have unequal in measure , then the greatest angle in measure is ………
5	The smallest angle of a triangle (in measure) is opposite to ………
6	In any triangle the greatest angle in measure is opposite to ………
7	If ABC is an obtuse-angled triangle at C , then AB ……… BC
8	If : $x > y$, z is positive number then : $x z >$ ………
9	Δ ABC in which : m ($\angle$ A) = 100° , then the greatest side in length is ………
10	The longest side in the triangle ABC in which m ($\angle$ B) = 105° is ……………
11	Δ ABC in which m ($\angle$ A) = 110° , then the greatest side in length is ……………
12	Δ ABC in which : m ($\angle$ C) = 112° , then the longest side is ……………
13	In Δ ABC : If m ($\angle$ B) = 120° , then the longest side in Δ ABC is ………
14	In Δ DEF if m ($\angle$ E) = 125° , then the longest side in this triangle is ………
15	In Δ ABC : If m ($\angle$ A) = 130° , then the longest side is ………
16	In triangle ABC , if m ($\angle$ A) = 70° , m ($\angle$ B) = 30° , then the longest side in length is ………
17	In Δ ABC : if m ($\angle$ A) = 67° and m ($\angle$ B) = 33° , then AB > ……… > ………
18	Δ ABC in which m ($\angle$ B) = 70° and m ($\angle$ C) = 35° the longest side in length is ………

19	In Δ ABC , m ($\angle$ A) = 50° , m ($\angle$ B) = 65° , then the number of axes of symmetry equals ……….
20	In Δ ABC if : m ($\angle$ A) = 50° and m ($\angle$ B) = 60° , then the longest side in this triangle is ……………
21	In Δ ABC m ($\angle$ B) = 70° and m ($\angle$ C) = 60° then AC …………… AB
22	In the isosceles triangle if : AB = AC , m ($\angle$ A) = 70° , then AB < ……………
23	In the triangle ABC : if m ($\angle$ B) – m ($\angle$ A) > m ($\angle$ C) , then AC ……… AB

[2] Essay problems:

1 ABC is a triangle in which : m ($\angle$ A) = 40° and m ($\angle$ B) = 75°

Order the lengths of the sides of the triangle descendingly.

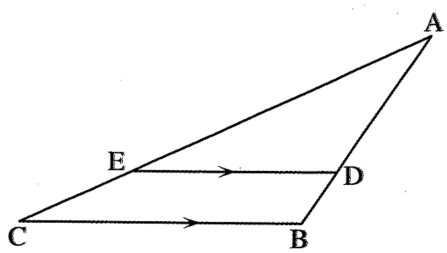
2 **In the opposite figure :**

ABC is an obtuse-angled triangle at B

, $\overline{DE}$ // $\overline{BC}$

Prove that :

AE > AD



3 ABC is a right-angled triangle at B , D $\in \overline{AC}$ and E $\in \overline{BC}$ where AD = BE

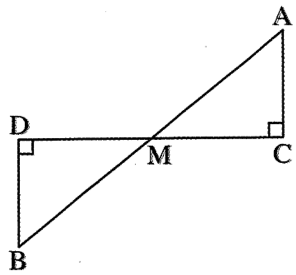
Prove that : m ($\angle$ CED) > m ($\angle$ CDE)

4 **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{M\}$, $\overline{AC} \perp \overline{CD}$ and $\overline{BD} \perp \overline{CD}$

Prove that :

AB > CD



Homework

[1] Choose the correct answer:

1	Δ XYZ , m ($\angle$ X) = 60° , m ($\angle$ Y) = 40° , then XZ .... XY (a) < (b) > (c) = (d) nothing.
2	ABC is a triangle in which : m ($\angle$ B) = 70° , m ($\angle$ C) = 50° , then AC .... AB (a) > (b) < (c) = (d) ≡
3	In a triangle ABC : m ($\angle$ B) = 75° , m ($\angle$ C) = 50° , then BC .... AB (a) < (b) > (c) = (d) ≡
4	ABC is a triangle in which : m ($\angle$ B) = 80° , m ($\angle$ C) = 50° , then BC .... AB (a) > (b) < (c) = (d) ≡
5	If : m ($\angle$ A) = 50° and m ($\angle$ B) = 60° in triangle ABC then AB ....AC (a) > (b) < (c) = (d) ≤
6	In a triangle ABC : If m ($\angle$ A) = 80° , m ($\angle$ C) = 60° , then AB .... BC (a) > (b) < (c) = (d) ≥
7	Triangle ABC : If m ($\angle$ B) = 70° , m ($\angle$ C) = 60° , then BC .... AB (a) < (b) > (c) = (d) ≥
8	In Δ LMN , if m ($\angle$ N) = 75° , m ($\angle$ M) = 60° , then LM .... LN. (a) > (b) < (c) = (d) twice
9	Δ ABC , m ($\angle$ B) = 90° , then AB .... AC (a) > (b) = (c) < (d) ≥
10	In Δ XYZ : If m ($\angle$ X) = 30° and m ($\angle$ Y) = 80° , then .... (a) XY < XZ (b) XY > XZ (c) XY = XZ (d) XY < YZ

11 In ΔABC : m ($\angle A$) = 60° and m ($\angle C$) = 45° , then ………

(a) AB < AC (b) AB = AC (c) AB > AC (d) $\overline{AB} \equiv \overline{AC}$

12 KLM is a triangle in which m ($\angle K$) = 50° and m ($\angle M$) = 60° , then which of the following statement is true ? ……………

(a) KL = KM (b) KM > KL (c) KM < ML (d) LM > KL

13 The triangle in which the measure of two angles are 74° and 53° is …………… triangle.

(a) a right-angled (b) an isosceles (c) an equilateral (d) a scalene

14 In ΔABC : If AB > AC , m ($\angle C$) = 70° , then m ($\angle B$) may equal ………

(a) 70° (b) 50° (c) 80° (d) 75°

15 In ΔABC , if m ($\angle A$) = 50° and m ($\angle B$) = 30° , then the shortest side in the triangle ABC is ……………

(a) $\overline{AB}$ (b) $\overline{CB}$ (c) $\overline{AC}$ (d) $\overleftrightarrow{BC}$

16 ΔABC which : m ($\angle A$) = 50° , m ($\angle B$) = 60° the longest side of it is ………

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{BC}$ (d) $\overline{CB}$

17 In ΔABC if : m ($\angle B$) = 60° and m ($\angle C$) = 50° , then the shortest side in triangle ABC is ……………

(a) $\overline{AC}$ (b) $\overleftrightarrow{BC}$ (c) $\overline{BC}$ (d) $\overline{AB}$

18 In the triangle ABC , if m ($\angle B$) = 90° , then the greatest side in length is ……………

(a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$ (d) $\overline{XY}$

19 In ΔABC if : m ($\angle B$) = 130° , then the longest side of it is ……………

(a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) it's median

20 In the triangle ABC : If m ($\angle B$) > m ($\angle C$) , then AB ……… AC

(a) < (b) > (c) = (d) otherwise

21 In ΔABC : if m ($\angle B$) > m ($\angle C$) , then AC ……… AB

(a) > (b) < (c) = (d) ≤

22	In Δ ABC , if m ($\angle$ B) > m ($\angle$ C) , then .... (a) AB < AC (b) AB = AC (c) AB > AC (d) $\overline{AB} \equiv \overline{AC}$
23	In Δ ABC : m ($\angle$ A) < m ($\angle$ C) < m ($\angle$ B) , then .... (a) AB > AC (b) BC > AC (c) AC > AB (d) BC > AB
24	The triangle ABC is obtuse-angled triangle at B , then the longest side is .... (a) AB (b) BC (c) AC (d) AD
25	Δ XYZ is right-angled at Y , then XZ .... YZ (a) = (b) > (c) ≤ (d) <
26	In Δ ABC : m ($\angle$ B) + m ($\angle$ C) = 3 m ($\angle$ A) , then m ($\angle$ A) = ...° (a) 30 (b) 60 (c) 45 (d) 90

[2] Essay problems:

1

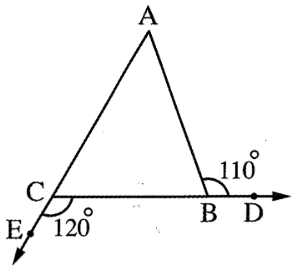
In the opposite figure :

ABC is a triangle , D $\in \overrightarrow{CB}$,

E $\in \overrightarrow{AC}$, m ($\angle$ ABD) = 110°

and m ($\angle$ BCE) = 120°

Prove that : AB > BC



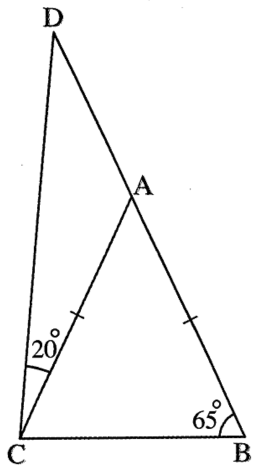
2

In the opposite figure :

AB = AC , m ($\angle$ ABC) = 65°

, m ($\angle$ ACD) = 20° , A $\in \overline{BD}$

Prove that : AB > AD



3

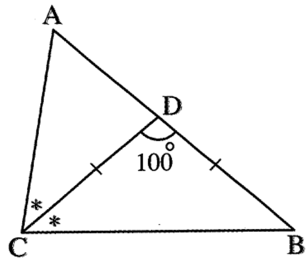
In the opposite figure :

ABC is a triangle , $\overrightarrow{CD}$ bisects ∠ C and intersects $\overline{AB}$ at point D

, m (∠ BDC) = 100° and DB = DC

Prove that :

AC > DB



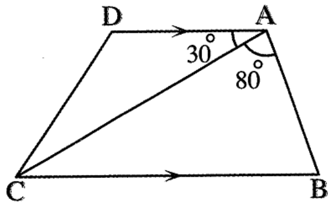
4

In the opposite figure :

$\overrightarrow{AD}$ // $\overline{BC}$, m (∠ BAC) = 80° and m (∠ DAC) = 30°

Prove that :

BC > AB

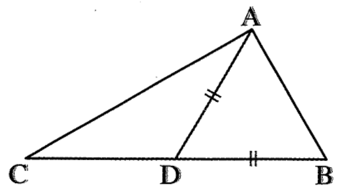


5

In the opposite figure :

ABC is a triangle and D ∈ $\overline{BC}$ where BD = AD

Prove that : BC > AC



Sheet (8)
Triangle inequality

Generally

In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.

Generally

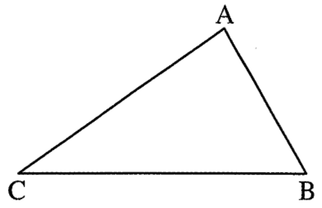
In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.

i.e. In any triangle such as ΔABC

, we get : $AB + BC > AC$

$, BC + CA > AB$

$, CA + AB > CB$



Corollary

The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum.

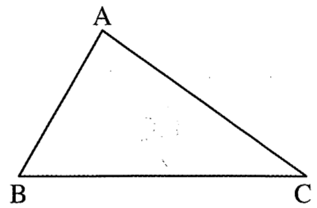
And you can prove that from the triangle inequality as follows :

In any triangle ABC :

$AC + AB > BC$ (1)

$, \because AB + BC > AC$ **i.e.** $BC > AC - AB$ (2)

From (1) and (2) , we deduce that : $\boxed{AC - AB < BC < AC + AB}$



Remark

To check the possibility that three lengths can be side lengths of a triangle , do as follows :

Compare the greatest length with the sum of the other two lengths :

- If the greatest length is greater than or equal to the sum of the other two lengths , you deduce that the three given lengths couldn't be lengths of the three sides of a triangle. (**i.e.** no triangle could be drawn with these side lengths).
- If the greatest length is less than the sum of the other two lengths , you deduce that the three given lengths could be lengths of the three sides of a triangle. (**i.e.** a triangle could be drawn with these side lengths).

[1] Complete:

1	In any triangle the sum of any two sides ……… the length of the third side.
2	In Δ ABC , AB + BC – AC > ………
3	The length of any side in a triangle ……… the sum of the lengths of the two other sides.
4	ABC is a triangle , if AB = 3 cm. and BC = 5 cm. , then AC $\in$] ……… , ……… [
5	Δ XYZ in which XY = 4 cm. and YZ = 3 cm. , then XZ $\in$] …………… , ……………[
6	If x cm. , 4 cm. and 5 cm. are lengths of the sides of a triangle , then ……… < x < ………
7	If : x , 8 , 7 cm. are lengths of the sides of a triangle then …………… < x < ……………
8	If the lengths of two sides in triangle are 3 cm. and 9 cm. , then …………… < the length of third side < ……………
9	If the length of two sides of an isosceles triangle are 4 cm. and 10 cm. , then the length of the third side is ………
10	If the length of two sides in an isosceles triangle are 3 cm. and 7 cm. , then the length of the third side = ……… cm.
11	The length of two sides in an isosceles triangle are 8 cm. , 4 cm. then the length of the third side = ……………
12	If the lengths of two sides in an isosceles triangle are 6 cm. , 3 cm. , then the lengths of the third side is …………… cm.
13	The length of two sides in the isosceles triangle are 3 cm. and 8 cm. , then the length of third side equals ……… cm.
14	The triangle whose side lengths are $(2x - 1)$ cm. , $(x + 3)$ cm. , and 7 cm. becomes an equilateral triangle when x = …………… cm.

Homework

[1] Choose the correct answer:

1. The sum of lengths of any two sides in any triangle .... the length of the third side.
(a) is less than (b) is greater than (c) equals (d) otherwise

2. If the lengths of two sides in an isosceles triangle are 2 cm. and 5 cm. , then the length of the third side is .... cm.
(a) 2 (b) 3 (c) 5 (d) 7

3. Δ ABC , AB = 2 cm. , BC = 7 cm. , then AC may equal ....
(a) 2 cm. (b) 5 cm. (c) 9 cm. (d) 8 cm.

4. The numbers 6 , 3 , ... can be lengths of sides of an isosceles triangle.
(a) 3 (b) 6 (c) 9 (d) 11

5. If the lengths of two sides in the isosceles triangle are 3 cm. , 7 cm. , then the length of third side = ...
(a) 3 cm. (b) 7 cm. (c) 10 cm. (d) 4 cm.

6. The numbers 7 , 3 and .... can be lengths of sides of an isosceles triangle.
(a) 3 (b) 10 (c) 7 (d) 4

7. If 3 cm. , 7 cm. are two side lengths in a triangle , then the smallest number of third side is ....
(a) 3 cm. (b) 4 cm. (c) 5 cm. (d) 6 cm.

8. The numbers 5 , 4 and .... can be lengths of sides of a triangle.
(a) 8 (b) 9 (c) 10 (d) 12

9. The numbers 4 , 8 , ... can be lengths of sides of an isosceles triangle.
(a) 4 (b) 8 (c) 12 (d) 3

10. If any sides in isosceles triangle 8 cm. , 4 cm. , then the length of the third side is ....
(a) 8 (b) 4 (c) 12 (d) 5

11 The lengths of two sides in a triangle are 4 cm. and 9 cm. and it has on axis of symmetry , then the length of third side is ....

(a) 4 cm. (b) 5 cm. (c) 9 cm. (d) 13 cm.

12 The lengths of 5 cm. , 6 cm. and .... can be length of the sides of a triangle.

(a) 15 cm. (b) 13 cm. (c) 11 cm. (d) 8 cm.

13 The numbers 5 , 7 , ... can be lengths of sides of triangle.

(a) 12 (b) 3 (c) 2 (d) 13

14 3 , 10 , ... can be lengths of sides of an isosceles triangle.

(a) 10 (b) 8 (c) 6 (d) 4

15 In Δ ABC if : AB = 3 cm. and BC = 5 cm. , then AC $\in$ ....

(a) $]3 , 8]$ (b) $[2 , 8]$ (c) $]2 , 8[$ (d) $]2 , 5[$

16 In the triangle ABC , if BC = 9 cm. , AB = 7 cm. , then m ($\angle$ C) .... m ($\angle$ A)

(a) = (b) $\geq$ (c) > (d) <

17 Which of the following can be sides to draw the triangle ....

(a) 5 cm. , 6 cm. , 12 cm. (b) 5 cm. , 6 cm. , 11 cm.

(c) 5 cm. , 6 cm. , 4 cm. (d) 4 cm. , 6 cm. , 10 cm.

18 Which of the following numbers can be the lengths of sides of a triangle ?

(a) 4 , 6 , 10 (b) 4 , 6 , 8 (c) 2 , 3 , 6 (d) 4 , 5 , 10

19 The lengths which can be the lengths of the sides of a triangle are ....

(a) 3 , 4 , 7 (b) 3 , 3 , 6 (c) 3 , 5 , 7 (d) 1 , 5 , 7

20 Which of the following set of numbers can be lengths of sides of a triangle ....

(a) 2 , 3 , 6 (b) 2 , 3 , 5 (c) 2 , 3 , 4 (d) 2 , 3 , 7

21 How many different triangles can be formed with sides of lengths a whole number of cm. and each with perimeter 7 cm. ?

(a) 1 (b) 2 (c) 3 (d) 4

22 If the length of one side of a triangle is 5 cm. , then which of the following could be the lengths of the other two sides ?

(a) 2 cm. and 3 cm. (b) 7 cm. and 2 cm.

(c) 2 cm. and 2 cm. (d) 4 cm. and 6 cm.

23 Which of the following numbers cannot be the lengths of sides of a triangle ....

(a) 7 , 7 , 5 (b) 9 , 9 , 9 (c) 3 , 6 , 12 (d) 3 , 4 , 5

24 In any triangle ABC : AB .... BC – AC

(a) > (b) < (c) = (d) ≤

25 In the triangle ABC , AC .... (AB – BC)

(a) > (b) ≥ (c) ≤ (d) <

26 In any triangle ABC , AB + BC .... AC

(a) = (b) < (c) > (d) ≤

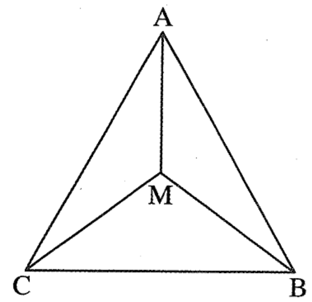
[2] Essay problems:

1 **In the opposite figure :**

ABC is a triangle in which M is a point inside it.

Prove that :

$MA + MB + MC > \frac{1}{2}$ the perimeter of the triangle ABC



2 Prove that the length of any side in a triangle is less than half of the perimeter.

3 Prove that the sum of the lengths of two diagonals in a convex quadrilateral is less than its perimeter.